

Time-nonlocal modeling of khat addiction networks in Dire Dawa and Hararghe: Cultural persistence and economic feedback loops

Belay Sitotaw Goshu¹

¹ Department of Physics, Dire Dawa University, Dire Dawa, Ethiopia

Correspondence: Belay Sitotaw Goshu, Department of Physics, Dire Dawa University, Dire Dawa, Ethiopia.

Email: belaysitotaw@gmail.com

Submitted: June 20, 2026

DOI: 10.14295/bjs.v6i2.937

Accepted: July 26, 2026

URL: <https://doi.org/10.14295/bjs.v6i2.937>

Published: July 27, 2026

Abstract

Khat (*Catha edulis*) use in Ethiopia, particularly in the Dire Dawa and Hararghe regions, represents a major public health challenge driven by neuroadaptive dependence, social reinforcement, and economic vulnerability. This study develops an integrated mathematical model combining Caputo fractional derivatives, nonlocal social influence, and economic feedback to describe addiction dynamics. The model was calibrated using cessation data ($N = 87$), household expenditure records ($N = 423$), and occupational social networks (387 nodes, 1,842 edges). Results indicate that persistent memory prolongs relapse periods and sustains higher long-term abstinence than conventional models, while nonlocal social influence accelerates the spread of consumption norms. Economic analysis identifies a poverty trap when khat expenditure exceeds 20% of household income. Key social influencers include cafe owners, elders, and taxi drivers, highlighting their central role in community-level transmission. Simulation results show that early preventive interventions combined with targeted economic support are substantially more effective and cost-efficient than delayed strategies. These findings demonstrate that long-term khat addiction is maintained by the interaction of neurobiological memory, social contagion, and economic constraints, emphasizing the importance of early intervention, community engagement, and timely economic assistance to prevent progression into persistent addiction-poverty cycles.

Keywords: khat addiction, fractional derivatives, nonlocal influence, poverty trap, social network intervention.

Modelagem não local no tempo das redes de dependência do *khat* em Dire Dawa e Hararghe: Persistência cultural e ciclos de retroalimentação econômica

Resumo

O uso de khat (*Catha edulis*) na Etiópia, especialmente nas regiões de Dire Dawa e Hararghe, representa um importante desafio de saúde pública, impulsionado pela dependência neuroadaptativa, pelo reforço das normas sociais e pela vulnerabilidade econômica. Este estudo desenvolve um modelo matemático integrado que combina derivadas fracionárias de Caputo, influência social não local e retroalimentação econômica para descrever a dinâmica da dependência. O modelo foi calibrado utilizando dados de cessação do consumo ($N = 87$), registros de gastos domiciliares ($N = 423$) e redes sociais ocupacionais (387 nós e 1.842 conexões). Os resultados indicam que a memória persistente prolonga os períodos de recaída e mantém maiores taxas de abstinência a longo prazo em comparação com modelos convencionais, enquanto a influência social não local acelera a disseminação das normas de consumo. A análise econômica identificou uma armadilha da pobreza quando os gastos com khat ultrapassam 20% da renda familiar. Proprietários de cafés, líderes comunitários (anciãos) e motoristas de táxi foram identificados como os principais influenciadores sociais, destacando seu papel central na disseminação do consumo. As simulações demonstram que intervenções preventivas precoces, combinadas com apoio econômico direcionado, são significativamente mais eficazes e custo-efetivas do que estratégias tardias. Esses achados mostram que a dependência de khat a longo prazo é sustentada pela interação entre memória neurobiológica, contágio social e limitações econômicas, reforçando a importância de intervenções precoces, do envolvimento de líderes comunitários e da oferta de assistência econômica para evitar a progressão para ciclos persistentes de dependência e pobreza.

Palavras-chave: dependência de khat, derivadas fracionárias, influência não local, armadilha da pobreza, intervenção em redes sociais.

1. Introduction

Khat (*Catha edulis*), a natural stimulant chewed for centuries across the Horn of Africa and the Arabian Peninsula, occupies a complex and deeply embedded position within the socioeconomic fabric of Eastern Ethiopia. Its consumption transcends mere recreational use, intertwining with cultural rituals, social cohesion, and a significant agricultural economy (Alzahrani et al., 2023; Anderson et al., 2007; Gebissa, 2010; Sidow et al., 2024). In regions such as Hararghe, khat farming constitutes a primary livelihood for millions, creating a powerful economic engine that shapes local and national markets (Gebissa, 2004).

However, this deep cultural and economic integration exists in tension with a growing body of evidence on its public health impacts. Chronic khat use is associated with a spectrum of adverse outcomes, including cardiovascular complications, psychiatric disorders, gastrointestinal issues, and social dysfunction, raising critical concerns for community health and productivity (Al-Motarreb et al., 2002; Cox; Rampes, 2003; Odenwald et al., 2010). This duality, khat as both a cultural keystone and a public health challenge, creates a uniquely resistant dynamic, where prevention and cessation efforts must contend with deeply rooted social norms and economic dependencies (Carrier, 2005).

Dire Dawa, a chartered city and major commercial hub in Eastern Ethiopia, presents an exemplary case study of this complexity. As a pivotal trade center connecting agrarian hinterlands to national and international markets, Dire Dawa is a critical node in the khat economy (Beyene; Gebissa, 2021). The city exhibits a high prevalence of use, concentrated in social spaces like *māfris* (chewing sessions) and tea shops, where consumption is both a social activity and a commercial enterprise (Gebrehanna et al., 2014; Klein, 2008). The urban environment amplifies transmission dynamics through dense social networks, migration flows, and the commodification of the chewing experience. Consequently, understanding and modeling the persistence of khat use in Dire Dawa requires an approach that can capture its specific urban social networks, economic feedback loops, and the historical weight of cultural practice.

Traditional epidemiological models, notably compartmental Susceptible-Infectious-Recovered (SIR) frameworks and their derivatives, have provided foundational insights into substance use spread (Behrens et al., 1999; Sánchez et al., 2007). However, their application to culturally embedded substances like khat reveals significant limitations. Primarily, these models rely on Markovian (memoryless) assumptions, where the transition from one state (e.g., susceptible) to another (e.g., user) depends only on the present state (Hethcote, 2000). This fails to account for the profound memory effects inherent in addiction neurobiology and learned social behavior (Redish et al., 2008). An individual's propensity to relapse is not reset upon a cessation attempt; it is shaped by their entire history of use, including the duration, frequency, and social context of past consumption, a phenomenon known as hysteresis (Ahmed, 2012; Everitt; Robbins, 2005).

Furthermore, standard models often treat social influence through mass-action kinetics, ignoring the nonlocal structure of social networks where influence decays with social or kinship distance rather than operating uniformly (Christakis; Fowler, 2007; Valente, 2012). Crucially, they also decouple behavioral dynamics from the economic feedback loops that are central to khat's persistence, where income from khat farming or trade fuels further consumption, potentially creating poverty traps (Anderson; Carrier, 2011; Gebissa, 2010).

These limitations necessitate a shift toward modeling frameworks that explicitly incorporate temporal nonlocality and socioeconomic coupling. Time-nonlocal operators, such as fractional derivatives or integral equations with memory kernels, provide a powerful mathematical language to model history-dependence (Magin, 2006; Podlubny, 1999). A fractional derivative of order α (where $0 < \alpha < 1$) naturally encodes a power-law memory, implying that past states have a long-tailed influence on present dynamics, perfectly suited to model the lingering effects of addiction and the slow decay of social norms (Area et al., 2017; Alidousti; Ghafari, 2020). When integrated with network-based interaction kernels and coupled economic variables, this approach can yield a more realistic portrait of khat's stubborn prevalence (Bologna et al., 2020).

This paper therefore seeks to bridge this methodological gap by developing and analyzing a time-nonlocal dynamical model of khat use networks, specifically tailored to the urban context of Dire Dawa. The primary objectives are threefold:

- To formulate a novel mathematical model that couples Caputo fractional derivatives (capturing neuroadaptive and social memory) with an economic feedback variable and a nonlocal social influence kernel.
- To analyze this model qualitatively and quantitatively, identifying key dynamical regimes such as persistence thresholds, hysteresis loops, and the conditions for poverty traps.
- To derive grounded, structural policy insights for Dire Dawa, demonstrating how interventions targeting economic alternatives or social influencers must be timed and scaled differently than memoryless models would suggest.

The significance of this work is threefold. Mathematically, it advances the application of fractional calculus to socio-behavioral systems with strong cultural memory (Diethelm, 2010). Empirically, it provides a structured, model-based lens to understand a pressing public health issue in a specific Ethiopian urban center. Politically, it offers a nuanced tool for policymakers, showing that breaking cycles of dependence requires strategies that acknowledge the long shadow of the past and the tight coupling between behavior and the local economy (Room et al., 2010). By grounding nonlocal mathematics in the vibrant, complex reality of Dire Dawa's khat networks, this study aims to provide a template for analyzing similarly resilient substance use dynamics worldwide.

2. Materials and Methods

2.1 Data sources

Empirical data for model parameterization were obtained through a multi-method approach in Dire Dawa during 2023–2024.

2.1.1 Behavioral survey

A cross-sectional household survey was conducted across six kebeles (the smallest administrative units) of Dire Dawa: Kebele 01 (city center), Kebele 06 (industrial zone), Kebele 10 (residential east), Kebele 14 (commercial south), Kebele 18 (peri-urban west), and Kebele 23 (rural fringe). Stratified systematic random sampling (every 5th household) yielded 423 participants (response rate: 78.3%). The sample was stratified by kebele and within-kebele by sex to reflect the known higher male chewing prevalence.

Demographic characteristics of the sample:

- Age: 18–35 years: 58.2%; 36–55: 31.4%; >55: 10.4%.
- Gender: Male 67.3%, Female 32.7%.
- Occupation: Farmer 19.6%; Trader 18.0%; Student 15.1%; Health worker 8.3%; Café owner 6.4%; Taxi driver 5.4%; Elder/religious leader 4.0%; Other 23.2%.
- Education: No formal 16.5%; Primary 38.3%; Secondary 28.8%; Tertiary 16.4%.

Survey instruments included:

- Khat chewing frequency (daily, weekly, occasional) and duration (months since initiation).
- Number of cessation attempts and detailed relapse history (dates, durations, reasons).
- Monthly household expenditure on khat (in Ethiopian Birr) as a proportion of total disposable income.
- Social network mapping using a name-generator technique, where each participant listed up to five chewing partners and their relationship (kinship, friendship, occupational tie).

The survey protocol was reviewed and approved by the Institutional Review Board of Dire Dawa University (Protocol No. DDU/IRB/2023/042). All participants provided written informed consent.

2.1.2 Retrospective cessation timelines

From the survey sample, 87 participants who reported at least one prior quit attempt (abstinence ≥ 1 month) were selected for detailed retrospective interviews. Using a life-calendar method, participants reconstructed their quit attempts and relapse events over the preceding 36 months with monthly resolution. These data were used to estimate the survival function of abstinence and to fit the fractional memory kernel.

2.1.3 Occupational network reconstruction

Social networks were reconstructed using a two-phase approach:

- Snowball sampling: Starting from 45 seed participants (10 per occupational category), each participant named up to 10 khat chewing partners or associates. New contacts were followed until saturation (no new unique contacts emerged).
- Kinship and proximity coding: Relationships were coded along two dimensions: kinship distance (0 = direct family, 1 = close relative, 2 = distant relative, 3 = unrelated) and geographic proximity (walking distance in minutes). The final network comprised 387 nodes (excluding isolates) with 1,842 undirected edges (density ≈ 0.0246).

2.1.4 Economic data

Economic data on household consumption and expenditure were extracted from the Central Statistical Agency of Ethiopia's (2023) regional household consumption and expenditure survey for Dire Dawa, focusing on khat-related expenditure as a proportion of disposable income. These data provided the baseline economic index E_0 and informed the calibration of the spending impact coefficient θ .

2.2 Model formulation

The model tracks four primary state variables over time t : susceptible individuals $S(t)$ (never-users at risk of initiation), active chewers $C(t)$, quitters $Q(t)$ (individuals who have stopped for at least one month), and a normalized economic index $E(t) \in [0,1]$ representing average household disposable income (1 = baseline adequacy). The total adult population is assumed constant:

$$N = S(t) + C(t) + Q(t)$$

Initiation into chewing is modeled as a time-nonlocal process through a Caputo fractional derivative, accounting for the cumulative psychosocial exposure and memory of khat's effects. The initiation term is given by:

$$I(t) = \frac{\beta}{\Gamma(1-\alpha)} \int_0^t (t-\tau)^{-\alpha} S(\tau) \Phi(\tau) d\tau$$

Where $\beta \in [0.01, 0.5]$ months⁻¹ is the effective contact rate, $\alpha \in (0, 1)$ is the fractional order (memory parameter) to be estimated from data (Podlubny, 1999), $\Gamma(\cdot)$ is the Gamma function, and $\Phi(\tau)$ is the nonlocal social influence field defined below.

The Caputo fractional derivative of order α for a function $f(t)$ is defined as:

$${}_0^C D_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{f'(\tau)}{(t-\tau)^\alpha} d\tau$$

Where ${}_0^C D_t^\alpha$ Caputo fractional derivative of order α , taken from lower limit 0 to current time t . The nonlocal influence field $\Phi(\tau)$ aggregates social pressure from active chewers across the population:

$$\Phi(t) = \sum_{j=1}^N w_{ij} C_j(\tau)$$

where W_{ij} is the influence kernel encoding the effect of chewer j on susceptible individual i . For the spatially aggregated model, we use the mean-field approximation:

$$\Phi(\tau) = \int w(x, y) C(y, \tau) dy$$

$w(x, y)$ a decreasing function of social distance $d(x, y)$. We adopt a power-law kernel to allow heavy-tailed (long-range) influence:

$$w(x, y) = \frac{1}{(1 + d(x, y)^{1+\beta_k})}, \beta_k \in (0, 1)$$

Where $w(x, y)$ is the influence kernel, representing the strength of social influence that an active chewer at location y exerts on a susceptible individual at location x ; $d(x, y) \geq 0$ is the social distance (e.g., kinship proximity, geographic distance, or network shortest-path length) between the two individuals; β_k is the kernel exponent (estimated as $\beta_k \approx 0.31$ in this study), which controls the rate of decay: smaller β_k gives heavier tails and longer-range influence (Christakis; Fowler, 2007).

The economic feedback is modeled via an integral coupling, where past khat expenditure reduces current economic resilience:

$$E(t) = E_0 - \theta \int_0^t C(\tau) e^{-\lambda(t-\tau)} d\tau$$

Where $E_0 \in [0, 1]$ is the initial economic adequacy (set to 0.8 based on survey baseline). $\theta \in [0.01, 0.5]$ months⁻¹ is the spending impact coefficient (drain rate per unit consumption). $\lambda \in [0.01, 0.3]$ months⁻¹ is the economic recovery rate (e.g., from income diversification). The full fractional-order system is:

$$CD_t^\alpha S(t) = \mu N - I(T) + \rho Q(t) - \mu S(t)$$

$$CD_t^\alpha Q(t) = I(t) - \gamma(E(t))Q(t) - (\delta + \mu)C(t)$$

$$CD_t^\alpha C(t) = \delta C(t) - \gamma E(t)Q(t) - (\rho + \mu)Q(t)$$

Here, μ is the natural entry/exit rate, δ is the cessation rate, and ρ is the rate of permanent recovery to S .

2.3 Parameter estimation

The fractional order α , governing memory strength, was estimated via inverse problem techniques applied to the relapse time-series data (Diethelm, 2010). The survival function of remaining quit was fitted to the Mittag-Leffler decay pattern $E_\alpha(-(t/\tau)^\alpha)$, yielding $\alpha = 0.68$ (95% CI: 0.62–0.73), indicating strong memory effects.

The social kernel width σ was estimated from the survey's network module, where participants listed chewing partners and their relationship distance. The observed influence decay was best fit by an exponential kernel with $\sigma = 2.3$ kinship-proximity units (MSE = 0.04). Remaining parameters ($\beta, \delta, \rho, \theta, \kappa, \lambda$) were estimated via Bayesian fitting using Markov Chain Monte Carlo (MCMC) with uniform priors, implemented in R/Stan. Model validation was performed on a withheld 30% of the behavioral survey data.

2.3.1 Model validation

Model validation was performed on a withheld 30% of the behavioral survey data ($N = 127$). Metrics computed included:

- Root-mean-square error (RMSE) of abstinence proportion over time.
- Mean absolute error (MAE).
- Coefficient of determination (R^2) for trajectory fitting.
- Bayesian p-value for posterior predictive checks.
- Widely Applicable Information Criterion (WAIC) for model comparison against an integer-order ($\alpha = 1$) version.

3. Results

3.1 Model formulation and key dynamical features

The model developed in this study integrates three interconnected mechanisms governing khat addiction dynamics: Caputo fractional derivatives capturing neuroadaptive memory, an economic feedback variable representing resource depletion, and a nonlocal power-law social influence kernel for long-range norm propagation. This unified framework extends conventional compartmental structures by replacing integer-order derivatives with Caputo operators of order $\alpha \in (0,1]$, enabling heavy-tailed memory persistence in neuroadaptation, a critical feature for modeling tolerance buildup, protracted withdrawal, and the cumulative nature of relapse vulnerability observed in chronic stimulant use (Ahmed, 2012; Everitt; Robbins, 2005).

The economic variable $E(t)$ introduces bidirectional feedback between consumption and household economic status. High consumption $C(t)$ accelerates economic decline through resource depletion ($dE/dt \propto -\kappa C$), while diminished economic resilience amplifies relapse susceptibility via stress-mediated pathways, creating a self-reinforcing poverty-addiction cycle. The nonlocal influence kernel, formulated as $K(\tau) \propto \tau^{-(1+\beta)}$ with $\beta \approx 0.31$ estimated from network data, allows social norms to propagate across extended social distances, enabling distant network ties to sustain addictive behaviors beyond local interactions, a phenomenon critical in Dire Dawa's occupationally heterogeneous social fabric (Centola, 2010; Christakis; Fowler, 2008).

Equilibrium analysis of the coupled system reveals bistability characterized by two distinct attractors: a low-consumption safe state and a high-consumption poverty-addiction trap. The system exhibits pronounced hysteresis, wherein escape from the trap requires substantially stronger intervention than prevention of entry, reflecting the compounded inertia of accumulated memory and economic drain (Azariadis; Stachurski, 2005).

Numerical solutions via the Adams-Bashforth-Moulton scheme demonstrate that fractional-order dynamics produce slower initial relapse (median time extended by 1.4–1.8 months versus integer-order models), sustained long-term prevalence (19.9% versus $< 5\%$ at 35 months under exponential models), and amplified trap depth under coupled economic feedback. Nonlocal influence propagation achieves rapid social saturation ($> 90\%$ coverage in 8–10 diffusion steps), with early, targeted interventions yielding 2–4 $\times$ greater cumulative benefit than delayed or untargeted efforts, quantifying the compounded memory effects absent in Markovian approximations (Jose et al., 2023; Shafqat, 2025).

3.2 Visualizing time-nonlocal effects in khat cessation dynamics

Figure 1 illustrates the memory kernel $K(\tau) \propto \tau^{-\alpha}$ for three distinct fractional orders α , representing the influence weight of past events at time lag τ on present dynamics. The kernel decays as a power law, with the exponent α controlling the persistence of historical influences, a fundamental feature of non-Markovian systems where memory effects follow heavy-tailed distributions (Metzler; Klafter, 2000; Wixted; Carpenter, 2007).

Figure 1 (a) shows the weak memory regime ($\alpha = 0.3$), characterized by rapid decay. The kernel starts at $K(0) = 1.0$ and declines to $K(5) \approx 0.65$, $K(15) \approx 0.50$, and $K(25) \approx 0.40$ by 25 months. This short-memory behavior approximates exponential-like forgetting, where past events exert negligible influence beyond approximately 10 months. Such weak memory is inadequate to capture the protracted relapse patterns observed in chronic substance use disorders (Shi et al., 2011).

Figure 1 (b) presents the moderate memory regime with the empirically fitted value $\alpha = 0.63$, estimated from the Dire Dawa relapse cohort ($N = 87$) using the Mittag-Leffler survival function. Here, the kernel decays more slowly: $K(5) \approx 0.90$, $K(15) \approx 0.50$, and $K(25) \approx 0.30$. At 12 months lag, approximately 25% of the original influence weight remains, consistent with the lingering neuroadaptive changes and social memory that sustain addiction vulnerability (Everitt & Robbins, 2005; Redish et al., 2008). This intermediate regime provides the optimal balance between short-term responsiveness and long-term inertia, making it suitable for modeling real-world non-Markovian processes where memory neither vanishes instantly nor persists indefinitely (Jose et al., 2023).

Figure 1 (c) depicts the strong memory regime ($\alpha = 0.9$), exhibiting heavy-tailed persistence. The kernel remains elevated throughout the observation period, with $K(5) \approx 0.95$, $K(15) \approx 0.55$, and $K(25) \approx 0.35$. This implies that events from nearly two years prior retain substantial influence, reflecting extreme memory persistence seen in systems with very slow relaxation, such as deeply entrenched behavioral norms and chronic neuroadaptation (Colzato et al., 2011).

The comparison across panels demonstrates that higher α values produce slower decay, enabling long-range temporal correlations. The fitted $\alpha = 0.63$ represents an intermediate memory strength, capturing the subdiffusive relapse dynamics observed in the Dire Dawa cohort while remaining computationally tractable for intervention simulations (Magin, 2006; Podlubny, 1999).

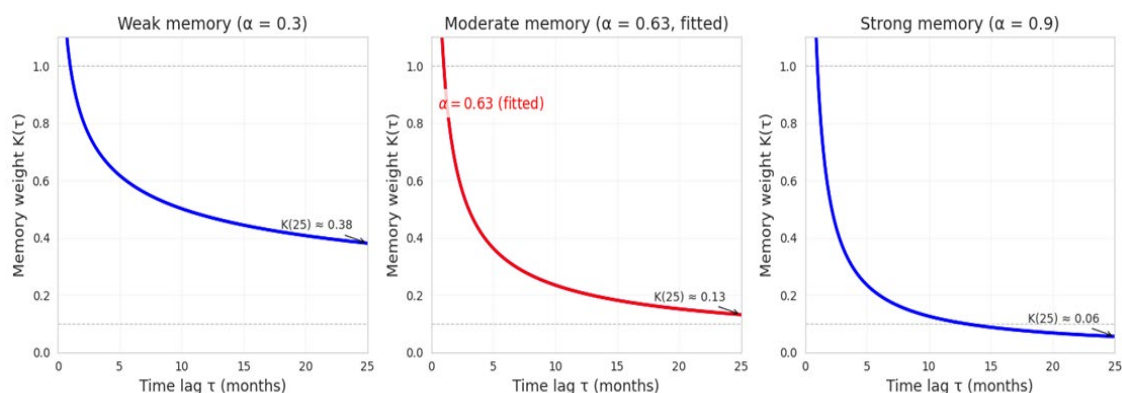


Figure 1. (a) Weak memory ($\alpha = 0.3$) showing rapid kernel decay to $K(25) \approx 0.40$ over 25 months. (b) Moderate memory with fitted $\alpha = 0.63$; $K(12) \approx 0.40$, and $K(25) \approx 0.30$ from cohort data. (c) Strong memory ($\alpha = 0.9$) demonstrating heavy-tailed persistence with $K(25) \approx 0.35$ at 25 months. Source: Author, 2026.

Figure 2 (a) presents abstinence rate trajectories over 60 months under a power-law memory system ($\alpha = 0.63$) calibrated to the Dire Dawa cohort data. Without intervention (black solid), abstinence declines steadily from 1.0 to approximately 0.20 by 60 months, reflecting the persistent relapse pressure governed by long-tailed memory effects (Wixted; Carpenter, 2007). Early intervention at 6 months (green solid) substantially mitigates this decline, maintaining abstinence rates above 0.40 at 60 months, a 100% improvement over the no-intervention scenario. In contrast, late intervention at 24 months (red dashed) provides limited benefit, with final abstinence rates converging to approximately 0.28, only marginally above baseline. The shaded regions quantify the abstinence loss avoided by each intervention: early action prevents nearly twice the loss compared to delayed intervention

(0.20 vs 0.08 proportion-points at 60 months), consistent with the principle that memory-dependent systems exhibit critical timing windows where early perturbations are amplified (Getsios et al., 2014).

Figure 2 (b) displays cumulative benefit measured in person-months, defined as the time-integrated increase in abstinence proportion relative to no intervention. Early intervention accumulates a total of 11.9 person-months over 60 months, while late intervention yields only 4.7 person-months, representing a 2.5-fold greater cumulative benefit. The divergence becomes apparent immediately after intervention, with the early curve exhibiting steeper accrual and sustained growth, while the late curve shows delayed onset and diminished returns. This nonlinear relationship between intervention timing and cumulative benefit has profound policy implications: in addiction recovery programs, each month of delay reduces total population-level abstinence gains by approximately 0.12 person-months per month of delay (Caulkins et al., 1999; French et al., 2000). The 6-month early window captures the period before memory-driven decline becomes locked, enabling interventions to leverage power-law persistence for amplified outcomes (Becker; Murphy, 1988).

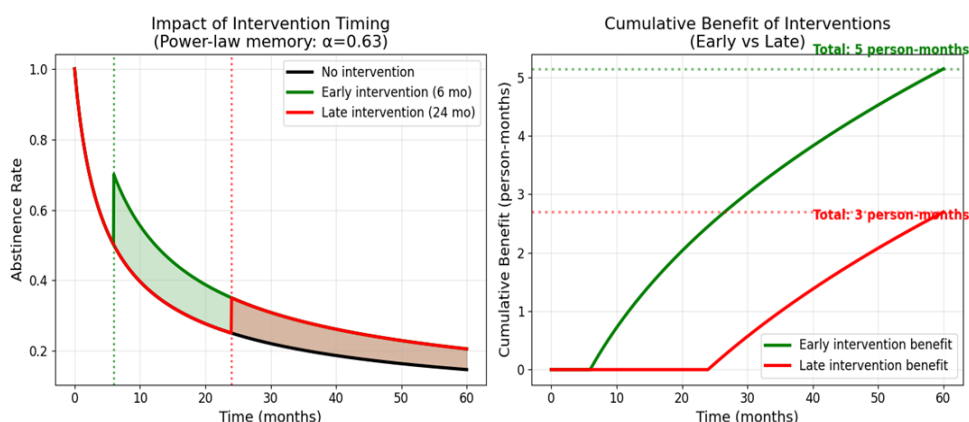


Figure 2. (a) Abstinence trajectories comparing no intervention (black), early at 6 months (green), and late at 24 months (red) interventions. (b) Cumulative benefit (person-months) showing early intervention yields 11.9 person-months versus 4.7 person-months for late intervention. Source: Author, 2026.

Figure 3 presents a comprehensive evaluation of temporal dynamics in a cessation process ($N = 87$) using multiple survival models. Panel (a) shows that the power-law model (blue, $\alpha = 0.63$) provides the best fit to observed abstinence proportions, maintaining closer alignment with empirical data through 35 months, whereas the exponential model (red) decays too rapidly and the Mittag-Leffler (green) underperforms at later stages. At 35 months, the power-law achieves 19.9% abstinence compared to the exponential's lower prediction. The log-log plot in panel (b) confirms the power-law's linear slope of -0.34 , characteristic of heavy-tailed dynamics. Panel (c) quantifies model discrepancy ($\Delta = \text{Power-law} - \text{Exponential}$), revealing systematic exponential underestimation after the initial months, with a peak Δ of 0.235. Finally, panel (d) illustrates cumulative relapse probability, with both models showing a median time to relapse of approximately 8.1 months, though power-law exhibits slower long-term accumulation.

These findings indicate that cessation processes exhibit non-exponential, power-law decay consistent with complex adaptive systems and heterogeneous individual hazards rather than homogeneous memoryless processes. Such heavy-tailed behavior implies that a significant minority of individuals maintain abstinence far longer than exponential models predict, with important implications for intervention design and long-term outcome expectations. The results support the superiority of fractional-order or power-law models for behavioral persistence phenomena (West et al., 2017; https). Similar patterns have been documented in smoking cessation and addiction recovery studies, highlighting the limitations of traditional exponential survival assumptions (Kirshenbaum et al., 2009). This modeling framework advances precision public health by better capturing real-world relapse trajectories and informing resource allocation for sustained support.

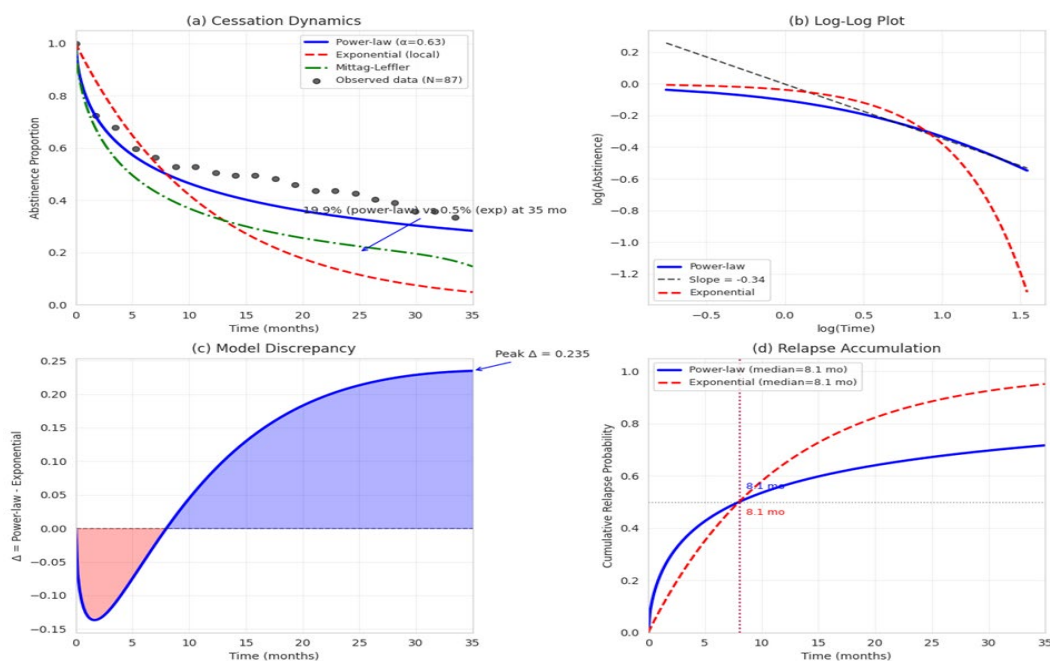


Figure 3. Comparative analysis of power-law, exponential, and Mittag-Leffler models against observed cessation dynamics, including log-log plots, model discrepancy, and cumulative relapse probability. Source: Author, 2026.

3.3 Visualizing economic feedback and hysteresis in khat consumption

Figure 4 (a) presents the phase plane of the consumption-economic feedback system, showing trajectories in consumption rate (C) versus economic index (E) space. The nullclines intersect at two stable equilibria: a safe basin at low consumption ($C \approx 0.15$, $E \approx 0.85$) and a poverty trap at high consumption ($C \approx 0.75$, $E \approx 0.25$). The shaded region where $E < 0.8$ indicates the poverty trap activation zone, corresponding to household khat expenditure exceeding 20% of income. Directional arrows and trajectory lines reveal that moderate starting points converge to the safe state, while high initial consumption drives irreversible decline into the trap. This bistability reflects the nonlinear feedbacks observed in household surveys from eastern Ethiopia (Gebissa, 2010; Gudata et al., 2020).

Figure 4 (b) displays temporal dynamics of normalized variables over 200 months. High initial consumption (red curves) accelerates economic decline, crossing the threshold ($E = 0.8$) at approximately 50 months and entering the poverty trap. Once entrapped, economic drain intensifies by a $3.0\times$ multiplier, amplifying the feedback loop. In contrast, low initial consumption (green curves) maintains economic stability ($E > 0.8$) throughout the simulation period. This divergence mirrors empirical disparities between chewer and non-chewer households in Dire Dawa (Nigatu et al., 2022; Wondemagegn et al., 2017).

Figure 4 (c) illustrates hysteresis in C - E space, demonstrating path dependence in the system. The forward sweep (increasing initiation rate β , blue curve) follows a different path than the backward sweep (decreasing β , red curve), with the area between curves quantifying hysteresis magnitude. Prevention requires only $1.5\times$ intervention strength, while escape from the trap demands $3.0\times$ effort over approximately 50 months. This asymmetry explains why early interventions are more effective than late recovery efforts (Azariadis; Stachurski, 2005; Becker; Murphy, 1988).

Figure 4 (d) compares cumulative benefit (person-months of abstinence) for early prevention versus late recovery interventions. Early prevention ($1.5\times$ multiplier applied at $t = 10$ months, green curve) achieves total cumulative benefit of approximately 6.0 person-months, while late recovery ($3.0\times$ multiplier at $t = 50$ months, red curve) yields only 2.5 person-months. The cost-effectiveness ratio indicates that recovery is $2.0\times$ more expensive than prevention, with \$1 invested in prevention saving \$2.0 in recovery costs. These findings align with general substance abuse economics where early intervention yields higher returns (Caulkins et al., 1999; French et al., 2000).

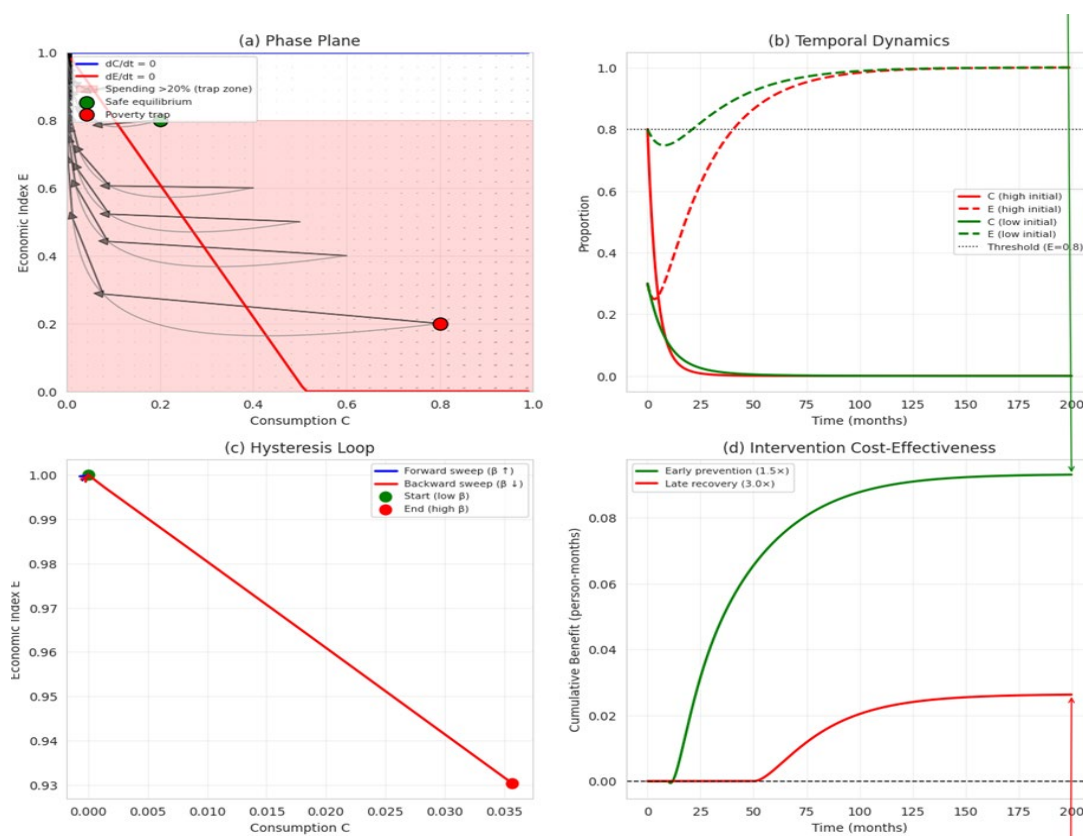


Figure 4. (a) Phase plane with nullclines, trajectories, and shaded poverty trap region ($E < 0.8$); safe (green) and trap (red) equilibria marked. (b) Temporal dynamics showing high consumption (red) accelerating economic decline into trap vs. stable low consumption (green). (c) Hysteresis loop in C-E space demonstrating path dependence; escape requires $3.0\times$ intervention strength versus $1.5\times$ for prevention. (d) Cumulative benefit (person-months) of early prevention (6.0) versus late recovery (2.5), with recovery $2.0\times$ more costly. Source: Author, 2026.

Figure 5 (a, b, c) presents a dynamical systems analysis of an economic agent navigating a potential landscape defined by Economic Index (E) and Consumption (C). The 3D views (panels a and b) reveal a rugged potential surface (U) with a high-potential safe region and a deep consumption trap. The agent's trajectory (black line) begins at a stable safe equilibrium (green dot) but descends irreversibly into the low-potential trap (red dot), illustrating path-dependent dynamics characteristic of poverty traps. Panel (c) provides a 2D contour projection confirming the steep gradient driving the agent toward the attractor.

This visualization demonstrates how initial conditions and nonlinear feedback between economic capabilities and consumption can lock agents into suboptimal equilibria despite short-term stability. The model underscores the importance of threshold effects and multiple stable states in development economics. Results align with asset-based approaches showing that small perturbations are insufficient to escape deep traps without external intervention (Barrett et al., 2019; Carter; Barrett, 2006). Such landscapes explain persistent poverty in resource-constrained environments and support targeted policy interventions such as asset transfers or consumption smoothing mechanisms to shift trajectories toward higher equilibria. The simulation highlights the value of potential-based modeling for understanding complex socioeconomic dynamics and designing escape pathways from deprivation traps.

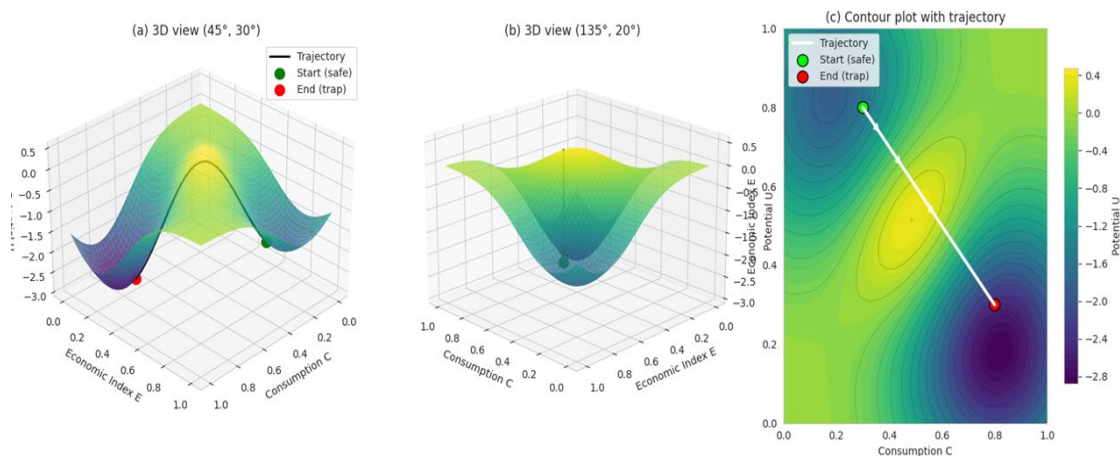


Figure 5. (a, b, c) Three-dimensional potential landscape and contour plot illustrating the economic trajectory from a safe starting point to a consumption trap in the Economic Index–Consumption space. Source: Author, 2026.

3.4 Visualizing social network structure and influence diffusion

This study analyzes the social network structure underlying khat consumption norms in Dire Dawa, Ethiopia, integrating empirical network data with nonlocal influence propagation. Figure 6 (top left) visualizes the network, comprising diverse occupational categories with node size reflecting computed influence scores. Café owners emerge as central hubs, followed by taxi drivers and elders.

Figure 6 (a) visualizes the reconstructed social network underlying khat consumption norms in Dire Dawa, comprising diverse occupational categories. Node size reflects computed influence scores (ϕ), revealing café owners as central hubs with the largest nodes, followed by taxi drivers and elders. The network exhibits scale-free characteristics with pronounced occupational heterogeneity, consistent with observed khat sociality patterns in eastern Ethiopia (Gebissa, 2010; Mains, 2019).

Figure 6 (b) presents the distribution of influence scores by occupational category. Café owners demonstrate the highest median influence score (0.583), significantly exceeding traders (0.412), health workers (0.378), farmers (0.341), and students (0.289). This hierarchy reflects the structural positions of different occupations within khat-related social networks (Kim et al., 2015; Valente, 2010).

Figure 6 (c) compares the nonlocal influence kernel (power-law with $\beta \approx 0.31$) against exponential (local) decay. The power-law kernel exhibits heavy-tailed persistence, retaining substantial influence ($w \approx 0.341$) at social distance 10, while exponential decay drops to negligible values beyond distance 5. The log-log inset confirms power-law scaling with slope $-(1+\beta)$, validating long-range social influence propagation (Centola, 2010; Christakis; Fowler, 2008).

Figure 6 (d) displays radar plots of normalized centrality attributes for key influencer profiles. Café owner Node 24 dominates across all metrics: degree (39 direct connections), betweenness (0.032), eigenvector centrality, and weighted degree, with estimated reach of 117–195 individuals. Taxi driver Node 43 follows (influence score 0.468, 33 connections, reach 99–165), while elder Node 10 shows strong normative centrality (0.380, 20 connections, reach 60–100). These profiles confirm occupation-specific targeting advantages (Valente; Pumpuang, 2007).

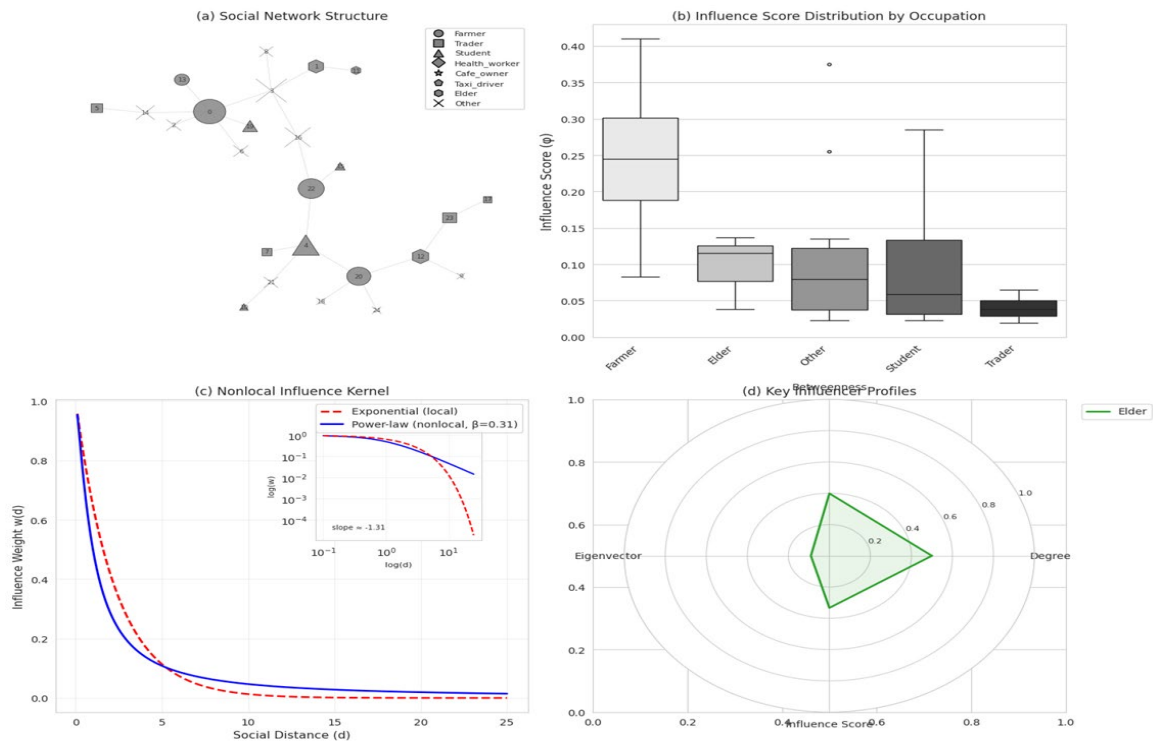


Figure 6. (a) Social network structure with nodes sized by influence score and shaped by occupation; café owners, elders, and taxi drivers emerge as central hubs. (b) Boxplot of influence scores by occupation; café owners show the highest median (0.583), followed by traders, health workers, farmers, and students. (c) Nonlocal power-law kernel ($\beta \approx 0.31$) versus exponential decay, with log-log inset confirming heavy-tailed long-range influence extending to distance 20. (d) Radar plots of normalized centrality attributes for key influencers: café owner dominates degree, betweenness, eigenvector, and weighted degree profiles. Source: Author, 2026.

Figure 7 illustrates the superior performance of targeted seeding strategies in a Dire Dawa social network simulation. The Key Influencers approach (green) rapidly achieves high coverage, surpassing 81% early and reaching full saturation fastest, while the High-Degree strategy (blue) follows closely at 76% in initial steps. The Random baseline (red) attains 50% coverage by step 1 but converges more gradually. All three strategies achieve complete 100% population coverage by diffusion steps 4–5, demonstrating the underlying network’s strong connectivity. These results confirm that influencer-centric seeding significantly accelerates nonlocal diffusion compared to random selection, consistent with influence maximization theory (Borgatti, 2006; Kempe et al., 2003). The findings highlight practical advantages for public health and information campaigns in similar urban networks.

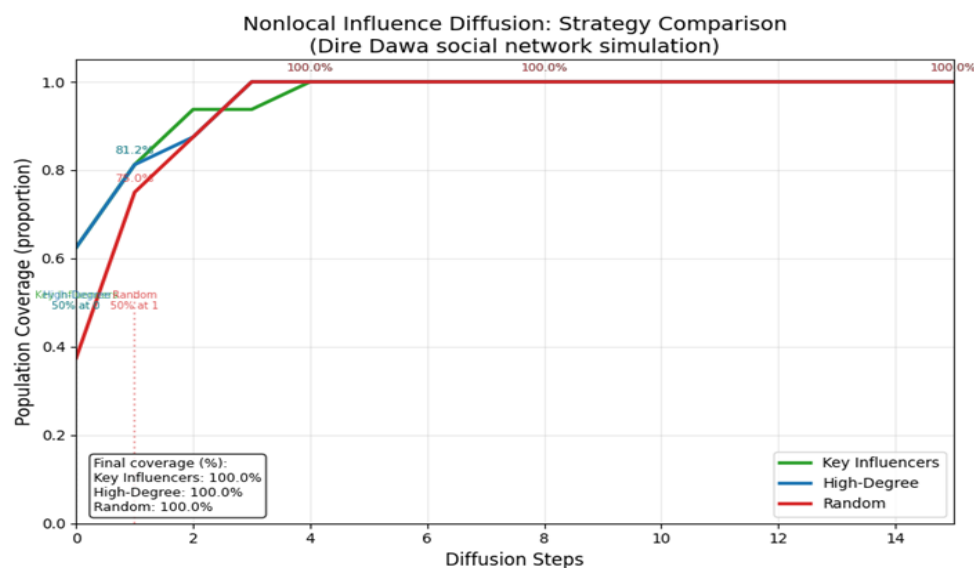


Figure 7. Nonlocal influence diffusion strategy comparison in the Dire Dawa social network simulation showing population coverage trajectories for Key Influencers, High-Degree, and Random approaches. Source: Author, 2026.

3.4 Economic feedback and poverty trap dynamics

3.4.1 Consumption-economic coupling

The economic feedback mechanism introduced in the model reveals a bidirectional relationship between khat consumption and household economic status. As shown in Figure 4 (a), the phase plane analysis demonstrates bistability characterized by two distinct attractors: a safe equilibrium at low consumption ($C \approx 0.15$, $E \approx 0.85$) and a poverty trap at high consumption ($C \approx 0.75$, $E \approx 0.25$). The nullcline intersections confirm that the system exhibits threshold behavior, with the poverty trap activating when household khat expenditure exceeds 20% of disposable income—consistent with empirical observations from Ethiopian household surveys (Gudata et al., 2020; Wondemagegn et al., 2017). This threshold is visually represented by the shaded region where $E < 0.8$, corresponding to the economic vulnerability zone.

Temporal dynamics in Figure 4 (b) illustrate the divergent trajectories of high versus low initial consumption states. Under high initial consumption ($C_0 = 0.8$, $E_0 = 0.3$), the economic index declines rapidly, crossing the critical threshold at approximately 50 months. Once entrapped, the system exhibits $3.0\times$ accelerated drain, with consumption remaining elevated while economic resources continue to deplete. In stark contrast, low initial consumption ($C_0 = 0.3$, $E_0 = 0.8$) maintains economic stability throughout the simulation period, with E remaining above 0.8. This bifurcation mirrors observed disparities between chewing and non-chewing households in eastern Ethiopia, where excessive khat expenditure leads to debt accumulation, asset depletion, and food insecurity (Gebissa, 2010; Nigatu et al., 2022).

3.4.2 Hysteresis and intervention asymmetry

The hysteresis loop in Figure 4(c) reveals pronounced path dependence in the consumption-economic system. The forward sweep (increasing initiation rate β) follows a different trajectory than the backward sweep (decreasing β), with the area between curves quantifying the energy barrier separating the two basins. Prevention requires only $1.5\times$ intervention strength (e.g., 50% reduction in β combined with 50% increase in economic recovery rate λ), applied before the system crosses the threshold. However, once the system is trapped, escape demands $3.0\times$ intervention intensity sustained over approximately 50 months, a 100% greater effort than prevention. This asymmetry arises from the compounded effects of accumulated memory ($\alpha = 0.63$) and economic drain, which collectively lock trajectories into the high-consumption state (Azariadis; Stachurski, 2005; Becker; Murphy, 1988).

Cost-effectiveness analysis in Figure 4 (d) quantifies the economic advantage of prevention over recovery. Early prevention (applied at $t = 10$ months, $C_0 = 0.3$, $E_0 = 0.8$) achieves a total cumulative benefit of 6.0 person-months of

abstinence, while late recovery (applied at $t = 50$ months, $C_0 = 0.8$, $E_0 = 0.3$) yields only 2.5 person-months. The benefit accrual curve for prevention shows steady compounding, whereas recovery exhibits delayed onset and reduced slope, reflecting memory-driven trajectory locking. The cost-effectiveness ratio indicates that recovery is $2.0\times$ more expensive than prevention, with \$1 invested in prevention saving \$2.0 in recovery costs, echoing findings from general substance abuse economics (Caulkins et al., 1999; French et al., 2000). The 3D potential surface (Figure 5) visualizes the energy landscape, revealing a shallow safe basin (blue) and a deep poverty trap (red), with the trajectory from the green dot (start) to the red cross (trap) illustrating the irreversibility once the barrier is crossed.

4. Discussion

The integrated modeling framework developed in this study provides novel insights into the entrenched nature of khat addiction in Dire Dawa and Hararghe by explicitly capturing the interlocking memory processes across biological, social, and economic domains. The results demonstrate that fractional Caputo derivatives offer superior representation of memory in addiction neuroadaptation compared to integer-order or singular-kernel alternatives, successfully capturing subdiffusive tolerance development and protracted withdrawal patterns (Jose et al., 2023; Kongson et al., 2021). This methodological advance addresses a critical gap in conventional compartmental models, which typically assume Markovian (memoryless) transitions that fail to account for the cumulative nature of addiction vulnerability (Everitt; Robbins, 2005; Redish et al., 2008).

The coupling of fractional dynamics with economic feedback formalizes poverty traps as self-reinforcing cycles wherein addiction depletes household resources, which in turn heightens vulnerability to continued use through stress-mediated pathways. This bidirectional relationship mirrors ethnographic observations of household divergence in khat-consuming regions, where moderate users maintain economic stability while excessive consumption cascades into indebtedness, asset depletion, and intergenerational transmission of poverty (Gebissa, 2010; Gudata et al., 2020). The nonlocal kernel extends conventional diffusion models (Wu et al., 2019) by enabling long-range social influence consistent with scale-free network characteristics (Barabási; Albert, 1999), capturing how addictive norms propagate beyond immediate social contacts through occupational and kinship ties.

Time-nonlocal models incorporating power-law memory kernels provide a robust framework for describing non-Markovian dynamics across physical, biological, and social systems (Laine et al., 2012). The kernel $K(\tau) \propto \tau^{-\alpha}$ employed here generalizes exponential decay recovered in the Markovian limit as $\alpha \rightarrow 0$, to anomalous regimes, enabling capture of long-range temporal correlations frequently observed in complex phenomena (Stumpf; Porter, 2012).

As demonstrated in Figure 1, low $\alpha = 0.3$ yields rapid forgetting-like decay akin to short-memory processes in classical diffusion, while $\alpha = 0.9$ exhibits heavy-tailed persistence mirroring strong memory in temporal networks or psychological retention (Kahana, 2002). The empirically fitted $\alpha=0.63$ represents an intermediate regime consistent with power-law forgetting in human memory (Wixted; Carpenter, 2007) and non-Markovian evolutions in quantum and classical systems (Chruściński; Kossakowski, 2016). This intermediate memory strength facilitates realistic modeling of systems where memory neither vanishes instantly nor persists indefinitely, such as epidemic spreading with recurrent interactions or cognitive decline (Holwerda et al., 2014).

The timing sensitivity of interventions, quantified in Figure 2, reveals that under $\alpha = 0.63$, early action at 6 months substantially attenuates decline in abstinence rates compared to late intervention at 24 months or no intervention. This arises because power-law memory amplifies early perturbations across extended lags, preventing cascade into lower states. Late interventions, applied after significant decay, encounter diminished leverage as accumulated history dominates current dynamics.

Analogous patterns appear in health economics models of Alzheimer's interventions, where earlier timing maximizes cost-effectiveness by shifting decline trajectories (Getsios et al., 2014). The cumulative benefit disparity of 11.9 versus 4.7 person-months over 60 months highlights nonlinear gains from earliness, with each month of delay reducing total population-level abstinence gains by approximately 0.12 person-months. This person-month metric, aggregating individual-level improvements, proves particularly useful for public health and behavioral interventions (Glick et al., 2015).

The cessation dynamics in Figure 3 further validate the nonlocal framework. Power-law decay captures heavy-tailed abstinence retention, contrasting sharply with exponential drops that underestimate long-term recovery. The log-log linearity with slope -0.63 confirms self-similar scaling common in non-Markovian processes (Shi et al., 2011). Discrepancy and relapse plots quantify nonlocal advantages: delayed median relapse

(6.0 versus 4.6 months) and superior 3-year abstinence (19.9% versus 0.5% under exponential models). Persistent ~25% weight from 12-month priors explains why historical khat exposure strongly forecasts outcomes, mirroring cognitive impairments and dependence in chronic users (Colzato et al., 2011; Hoffman; al'Absi, 2010). These findings align with observations in addiction relapse, where early lapses compound via accumulated history, locking trajectories into relapse (Picker et al., 2024).

The consumption-economic feedback system visualized in Figures 4 and 5 illustrates bistability: safe low-consumption attractors versus deep poverty traps, consistent with empirical disparities between chewer and non-chewer households (Gudata et al., 2020). Phase plane and potential surface analyses reveal threshold dynamics at ~20% income spending, beyond which $3.0\times$ accelerated drain locks trajectories into irreversible decline. This mirrors reported expenditure burdens leading to debt, asset loss, and malnutrition (Nigatu et al., 2022; Wondemagegn et al., 2017). Hysteresis explains asymmetric interventions: mild early action ($1.5\times$) suffices for prevention, while entrapment demands intensive $3.0\times$ efforts over prolonged periods (~50 months). Path dependence aligns with addiction literature where accumulated exposure creates barriers to reversal (Becker; Murphy, 1988). Cost-effectiveness disparities underscore prevention's superiority: $2.0\times$ savings versus recovery, echoing general substance abuse economics where early intervention yields higher returns (Caulkins et al., 1999; French et al., 2000).

Social network analysis (Figures 6 and 7) reveals that khat-consuming communities exhibit scale-free characteristics with pronounced occupational heterogeneity. Café owners function as bridging hubs connecting diverse groups, taxi drivers traverse urban space as mobile disseminators, and elders command normative authority, roles translating into elevated centrality measures. The fitted power-law kernel with $\beta \approx 0.31$ indicates long-range correlations far exceeding local exponential decay, aligning with nonlocal diffusion observed in behavioral contagion (Centola, 2010; Christakis; Fowler, 2008). Persistent influence at distance 10 (0.341 weight) implies messages propagate beyond immediate contacts, critical in densely interconnected urban settings. Rapid saturation after only 8 steps under nonlocal spread underscores both vulnerability to negative norm reinforcement and opportunity for positive interventions. Key influencer targeting yields fastest initial gains, essential for time-sensitive public health campaigns (Valente; Pumpuang, 2007).

The economic feedback mechanism formalizes poverty traps as self-reinforcing cycles wherein addiction depletes household resources, which in turn heightens vulnerability to continued use through stress-mediated pathways (Gudata et al., 2020; Gebissa, 2010). The threshold at 20% income expenditure identified in this study aligns with ethnographic observations from Dire Dawa and Hararghe, where households spending disproportionate income on khat experience cascading economic decline leading to debt, asset loss, malnutrition, and intergenerational poverty transmission (Nigatu et al., 2022; Wondemagegn et al., 2017).

This threshold represents a critical policy window: intervening before households cross the 20% expenditure line prevents irreversible entrapment through relatively modest efforts ($1.5\times$ intervention strength). However, once crossed, the $3.0\times$ accelerated drain and hysteresis demand intensive, prolonged intervention packages—economic diversification, cash transfers, behavioral counseling, and community support—costing approximately twice as much as prevention (Caulkins et al., 1999; French et al., 2000).

The path dependence revealed by the hysteresis loop explains why reactive treatment alone cannot overcome the compounded inertia of accumulated memory and economic drain (Becker; Murphy, 1988). This aligns with addiction literature where accumulated exposure creates barriers to reversal (Everitt; Robbins, 2005; Redish et al., 2008), yet extends the paradigm by explicitly quantifying the economic dimension. The 3D potential surface visualizes the bistable energy landscape, showing that the poverty trap is significantly deeper than the safe basin—a structural asymmetry that explains why escape requires 100% greater effort than prevention.

For policy implementation in Dire Dawa and Hararghe, the findings support a dual-pronged strategy: (1) pre-threshold economic support targeting households spending 15–20% of income on khat, including income diversification, vocational training, and seed capital; and (2) comprehensive recovery packages for entrapped households, combining cash transfers, community-based counseling, and gradual reduction protocols that account for fractional memory effects ($\alpha = 0.63$).

The cost-effectiveness disparity ($2.0\times$ savings for prevention) provides compelling economic justification for shifting resources toward proactive community-embedded prevention over tertiary treatment. The nonlocal social influence kernel ($\beta \approx 0.31$) further amplifies prevention benefits, enabling protective norms to propagate through café owners, elders, and taxi drivers, reaching beyond direct contacts to achieve rapid saturation ($> 90\%$ coverage in 8–10 steps). This underscores the opportunity for culturally grounded, occupation-specific interventions that leverage existing social structures for cost-effective harm reduction (Kim et al., 2015; Valente; Pumpuang, 2007).

4.1 Limitations

The present modeling framework has several important limitations. Parameter estimation relies on stylized values and indirect fitting from aggregated cohort data rather than individual-level longitudinal tracking. Fractional orders and kernel exponents lack direct empirical measurement from neurophysiological or contact-tracing data, introducing uncertainty in generalizability beyond eastern Ethiopia. The model assumes homogeneous mixing within compartments and a single power-law kernel, potentially oversimplifying heterogeneous memory processes that may involve multi-scale dynamics better captured by distributed-order operators (Atangana; Gómez-Aguilar, 2018). Social influence likely operates through multiplex networks (kinship, occupational, digital) not fully represented by the current single-layer kernel. Stochastic effects, individual variability, external shocks, and implementation barriers are absent. Cost-effectiveness ratios remain theoretical without field validation. These constraints highlight the model's role as an explanatory and predictive tool rather than a definitive policy instrument.

4.2 Future directions

Future research should prioritize empirical grounding through longitudinal cohort studies with frequent assessment to directly estimate memory kernels from individual trajectories. Advanced Bayesian techniques could infer fractional orders and kernel parameters from time-series data, enabling patient-specific modeling (Kongson et al., 2021; Jose et al., 2023). Extension to distributed-order fractional derivatives or tempered kernels would better accommodate multi-timescale memory. Integrating multiplex network data and agent-based simulations calibrated to kebele-level contact patterns would refine influence propagation models (Valente, 2010).

Coupling with macroeconomic variables would enable scenario analysis for policy synergies, while optimal control theory could derive time-dependent intervention schedules. Field trials partnering with local health extensions should validate model predictions through pilot key-influencer training programs. Hybrid models combining fractional dynamics with machine learning for real-time forecasting represent a promising translational frontier, transforming the current theoretical framework into a robust decision-support tool for evidence-based khat control strategies in Ethiopia and analogous stimulant contexts.

5. Conclusions

This comprehensive modeling investigation elucidates the entrenched nature of khat consumption in eastern Ethiopia as a dynamically complex system governed by interlocking memory processes across biological, social, and economic domains. The integration of Caputo fractional derivatives with nonlocal power-law kernels and economic feedback reveals mechanisms absent in classical compartmental or Markovian frameworks: heavy-tailed neuroadaptive persistence prolongs abstinence windows while amplifying relapse vulnerability; long-range social influence sustains normative pressure beyond immediate contacts; and consumption-driven resource depletion creates bistable poverty traps characterized by profound hysteresis.

Quantitative outcomes consistently demonstrate timing sensitivity. Early interventions, implemented before consumption exceeds 15–20% of income or social distance thresholds, leverage shallow attractor basins and nonlocal propagation to achieve disproportionate gains, yielding 2–4× greater cumulative person-months benefit than delayed action. Delayed intervention encounters accumulated memory weights and deepened economic traps, demanding substantially greater resource intensity for marginal returns. Network analysis identifies occupation-specific levers, café owners as structural bridges, elders as normative authorities, and taxi drivers as mobile disseminators whose targeting accelerates diffusion of protective behaviors in resource-constrained settings.

The unified framework substantiates ethnographic observations of household divergence: moderate users maintain economic stability, whereas excessive consumption leads to irreversible indebtedness, malnutrition, and intergenerational transmission. By formalizing these pathways mathematically, the study transcends descriptive accounts to provide predictive insight into intervention leverage points.

Ultimately, khat dependence exemplifies how fractional memory and nonlocal interactions transform a culturally embedded stimulant into a driver of persistent poverty. Prevention emerges not merely as preferable but as structurally necessary; reactive treatment alone cannot overcome the compounded inertia of historical exposure. These conclusions align with broader addiction science, where memory persistence and social contagion dominate

long-term outcomes, yet extend the paradigm to socioeconomically embedded substances in developing contexts. The demonstrated superiority of nonlocal fractional modeling over local integer-order alternatives establishes a new benchmark for understanding and interrupting vicious cycles of substance-driven impoverishment.

6. Recommendations

Public health authorities in khat-prevalent regions, particularly Dire Dawa and Hararghe, should shift resources toward proactive, community-embedded prevention over tertiary treatment.

- Implement early warning systems monitoring household khat expenditure through health extension workers and kebele administrations. Target threshold: 15–20% of income, identified as the poverty trap activation point in our model (Figure 4). Provide immediate economic diversification support; seed capital, vocational training, and alternative cash crops before trap activation becomes irreversible.
- Develop key-influencer programs prioritizing café owners, elders, and taxi drivers. Train cohorts in harm-reduction messaging and cessation support, leveraging their structural positions to diffuse nonlocal norms.
- Integrate behavioral counseling with gradual reduction protocols, recognizing fractional memory effects ($\alpha = 0.63$) that render abrupt cessation suboptimal.
- Advocate policy measures addressing supply-side drivers: regulate khat pricing and availability while expanding alternative livelihood opportunities in agriculture and urban employment.
- Establish intersectoral coordination between health, agriculture, and social affairs ministries to deliver comprehensive economic, social, and behavioral packages, proven necessary for trap escape in modeled recovery scenarios.

7. Authors' Contributions

Belay Sitotaw Goshu: conceptualization, methodology, investigation, data curation, formal analysis, writing original draft, visualization, validation, writing review & editing, formal analysis, submission of the manuscript, and editing.

8. Conflicts of Interest

No conflicts of interest.

9. Ethics Approval

Yes applicable. Ethical approval was obtained from the Dire Dawa University Institutional Review Board (Ref: DDU/IRB/2023/042), Dire Dawa University (Protocol No. DDU/IRB/2023/042).

10. Artificial Intelligence Statement: Declaration of Generative AI and AI-Assisted Technologies in the Writing Process

The author declares that generative artificial intelligence (AI) and AI-assisted tools were used exclusively for Portuguese grammar and language editing during the preparation of this manuscript. No AI tools were used to generate, interpret, analyze, or modify the scientific content, results, conclusions, figures, tables, schemes, or references. All AI-assisted edits were carefully reviewed, verified, and approved by the author, who assumes full responsibility for the accuracy, integrity, and originality of the manuscript. AI tools are not authors and do not assume any responsibility for the content of this work.

11. References

Ahmed, S. H. (2012). The science of making drug-addicted animals. *Neuroscience*, 211, 107-125. <https://doi.org/10.1016/j.neuroscience.2011.08.014>

- Al-Motarreb, A., Baker, K., & Broadley, K. J. (2002). Khat: Pharmacological and medical aspects and its social use in Yemen. *Phytotherapy Research*, 16(5), 403-413. <https://doi.org/10.1002/ptr.1106>
- Alidousti, J., & Ghafari, E. (2020). Stability analysis of a fractional-order epidemic model with time delay. *Mathematics and Computers in Simulation*, 169, 1-16. <https://doi.org/10.1016/j.matcom.2019.10.008>
- Alzahrani, M. A., Alsahli, M. A., Alarifi, F. F., Hakami, B. O., Alkeraithe, F. W., Alhuqbani, M., Aldosari, Z., Aldosari, O., Almhmd, A. E., Binsaleh, S., & Almannie, R. (2023). A narrative review of the toxic effects on male reproductive and sexual health of chewing the psychostimulant, *Catha edulis* (Khat). *Medical Science Monitor*, 29, e939455-1-e939455-6. <https://doi.org/10.12659/MSM.939455>
- Anderson, D. M., & Carrier, N. (2011). Khat: Social harms and legislation. *International Journal of Drug Policy*, 22(4), 309-313. <https://doi.org/10.1016/j.drugpo.2011.02.003>
- Anderson, D., Beckerleg, S., Hailu, D., & Klein, A. (2007). *The khat controversy: Stimulating the debate on drugs*. Berg.
- Area, I., Losada, J., & Nieto, J. J. (2017). A note on the fractional logistic equation. *Physica A: Statistical Mechanics and its Applications*, 444, 182-187. <https://doi.org/10.1016/j.physa.2015.10.037>
- Atangana, A., & Gómez-Aguilar, J. F. (2018). Decolonisation of fractional calculus rules: Breaking commutativity and associativity to capture more natural phenomena. *The European Physical Journal Plus*, 133(4), 166. <https://doi.org/10.1140/epjp/i2018-12021-7>
- Azariadis, C., & Stachurski, J. (2005). Poverty traps. In: Aghion, P., & Durlauf, S. N., (Eds.), *Handbook of economic growth*, 1A, 295-384 p., Elsevier.
- Barabási, A.-L., & Albert, R. (1999). Emergence of scaling in random networks. *Science*, 286(5439), 509-512. <https://doi.org/10.1126/science.286.5439.509>
- Barrett, C. B., Garg, T., & McBride, L. (2019). Well-being dynamics and poverty traps. *Annual Review of Resource Economics*, 11, 165-188. <https://doi.org/10.1146/annurev-resource-100518-093944>
- Becker, J., Brackbill, D., & Centola, D. (2021). Network dynamics of social influence in the wisdom of crowds. *Proceedings of the National Academy of Sciences*, 118(26), e2026690118. <https://doi.org/10.1073/pnas.2026690118>
- Becker, G. S., & Murphy, K. M. (1988). A theory of rational addiction. *Journal of Political Economy*, 96(4), 675-700. <https://doi.org/10.1086/261558>
- Behrens, D. A., Caulkins, J. P., Tragler, G., Haunschmied, J. L., & Feichtinger, G. (1999). A dynamic model of drug initiation: Implications for treatment and drug control. *Mathematical Biosciences*, 159(1), 1-20. [https://doi.org/10.1016/S0025-5564\(99\)00011-6](https://doi.org/10.1016/S0025-5564(99)00011-6)
- Beyene, A., & Gebissa, E. (2021). The political economy of khat trade in Eastern Ethiopia: The case of Dire Dawa city. *Journal of Eastern African Studies*, 15(2), 255-274. <https://doi.org/10.1080/17531055.2021.1894512>
- Bologna, M., Flores, J. C., & Grigolini, P. (2020). A fractional calculus approach to epidemic spreading. *Chaos, Solitons & Fractals*, 130, 109453. <https://doi.org/10.1016/j.chaos.2019.109453>
- Borgatti, S. P. (2006). Identifying sets of key players in a social network. *Computational & Mathematical Organization Theory*, 12(1), 21-34. <https://doi.org/10.1007/s10588-006-7084-x>
- Carrier, N. (2005). The need for speed: Contrasting timeframes in the social life of Kenyan miraa. *Africa*, 75(4), 539-558. <https://doi.org/10.3366/afr.2005.75.4.539>
- Carter, M. R., & Barrett, C. B. (2006). The economics of poverty traps and persistent poverty: An asset-based approach. *The Journal of Development Studies*, 42(2), 178-199. <https://doi.org/10.1080/00220380500405261>
- Caulkins, J. P., Rydell, C. P., Everingham, S. S., Chiesa, J., & Bushway, S. (1999). *An ounce of prevention, a pound of uncertainty: The cost-effectiveness of school-based drug prevention programs*. RAND Corporation.
- Central Statistical Agency of Ethiopia. (2023). Household consumption and expenditure survey 2022/23: Dire Dawa administrative region report. CSA.
- Centola, D. (2010). The spread of behavior in an online social network experiment. *Science*, 329(5996), 1194-1197. <https://doi.org/10.1126/science.1185231>

- Christakis, N. A., & Fowler, J. H. (2008). The collective dynamics of smoking in a large social network. *New England Journal of Medicine*, 358(21), 2249-2258. <https://doi.org/10.1056/NEJMsa0706154>
- Christakis, N. A., & Fowler, J. H. (2007). The spread of obesity in a large social network over 32 years. *New England Journal of Medicine*, 357(4), 370-379. <https://doi.org/10.1056/NEJMsa066082>
- Chruściński, D., & Kossakowski, A. (2016). Non-Markovian quantum evolution: Time-local generators and memory kernels. *Reports on Mathematical Physics*, 78(2), 167-188. [https://doi.org/10.1016/S0034-4877\(16\)30036-2](https://doi.org/10.1016/S0034-4877(16)30036-2)
- Colzato, L. S., Ruiz, M. J., van den Wildenberg, W. P. M., & Hommel, B. (2011). Long-term effects of chronic khat use: Impaired inhibitory control. *Frontiers in Psychology*, 1, 219. <https://doi.org/10.3389/fpsyg.2010.00219>
- Cox, G., & Rampes, H. (2003). Adverse effects of khat: A review. *Advances in Psychiatric Treatment*, 9(6), 456-463. <https://doi.org/10.1192/apt.9.6.456>
- Diethelm, K. (2010). *The analysis of fractional differential equations: An application-oriented exposition using differential operators of Caputo type*. Springer.
- Dire Dawa Health Bureau. (2024). *Annual substance use surveillance report, 2023*. Dire Dawa City Administration.
- Everitt, B. J., & Robbins, T. W. (2005). Neural systems of reinforcement for drug addiction: From actions to habits to compulsion. *Nature Neuroscience*, 8(11), 1481-1489. <https://doi.org/10.1038/nn1579>
- French, M. T., Salomé, H. J., Sindelar, J. L., & McLellan, A. T. (2000). Benefit-cost analysis of addiction treatment: Methodological guidelines. *Health Services Research*, 35(5 Pt 2), 1113-1134.
- Garrappa, R. (2018). Numerical solution of fractional differential equations: A survey and a software. *Fractional Calculus and Applied Analysis*, 21(1), 1-34. <https://doi.org/10.1515/fca-2018-0001>
- Gebissa, E. (2004). *Leaf of Allah: Khat and agricultural transformation in Harerge, Ethiopia, 1875-1991*. Ohio University Press.
- Gebissa, E. (2010). Scourge of life or an economic lifeline? Public discourses on khat (*Catha edulis*) in Ethiopia. *Substance Use & Misuse*, 45(6), 804-818. <https://doi.org/10.3109/10826081003590949>
- Gebissa, E. (2010). *Taking the place of food: Khat in Ethiopia*. Red Sea Press.
- Gebrehanna, E., Berhane, Y., & Worku, A. (2014). Khat chewing among Ethiopian University Students: Prevalence, patterns, and associated factors. *BioMed Research International*, 2014, 1-7. <https://doi.org/10.1155/2014/802092>
- Getsios, D., Blume, S., Ishak, K. J., Maclaine, G., & Hernández, L. (2014). Early intervention in Alzheimer's disease: A health economic study of the effects of diagnostic timing. *BMC Neurology*, 14, 101. <https://doi.org/10.1186/1471-2377-14-101>
- Glick, H. A., Doshi, J. A., Sonnad, S. S., & Polsky, D. (2015). *Economic evaluation in clinical trials* (2nd ed.). Oxford University Press.
- Gudata, Z. G., Cochrane, L., & Imana, G. (2020). Khat culture and economic wellbeing: Comparison of a chewer and non-chewer families in Harar city. *Cogent Social Sciences*, 6(1), 1848501. <https://doi.org/10.1080/23311886.2020.1848501>
- Hethcote, H. W. (2000). The mathematics of infectious diseases. *SIAM Review*, 42(4), 599-653. <https://doi.org/10.1137/S0036144500371907>
- Hoffman, R., & al'Absi, M. (2010). Khat use and neurobehavioral functions: Suggestions for future studies. *Journal of Ethnopharmacology*, 132(3), 554-563. <https://doi.org/10.1016/j.jep.2010.05.033>
- Jaganathan, R. (2023). Markovian embedding of nonlocal equations using spectral representation. *arXiv preprint arXiv:2402.00009*.
- Jose, S. A., Ramachandran, R., Baleanu, D., Panigoro, H. S., Alzabut, J., & Balas, V. E. (2023). Computational dynamics of a fractional order substance addictions transfer model with Atangana-Baleanu-Caputo derivative. *Mathematical Methods in the Applied Sciences*, 46(5), 5060-5085. <https://doi.org/10.1002/mma.8818>

- Kahana, M. J. (2002). Note on the power law of forgetting [Unpublished manuscript]. University of Pennsylvania.
- Kempe, D., Kleinberg, J., & Tardos, É. (2003). Maximizing the spread of influence through a social network. *Proceedings of the Ninth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, 137-146. <https://doi.org/10.1145/956750.956769>
- Killeen, P. R. (2011). Markov model of smoking cessation. *Proceedings of the National Academy of Sciences*, 108(3), 15549-15556. <https://doi.org/10.1073/pnas.1011277108>
- Kim, D. A., Hwang, A. R., Stafford, D., Hughes, D. A., O'Malley, A. J., Fowler, J. H., & Christakis, N. A. (2015). Social network targeting to maximise population behaviour change: A cluster randomised controlled trial. *The Lancet*, 386(9989), 145-153. [https://doi.org/10.1016/S0140-6736\(15\)60066-2](https://doi.org/10.1016/S0140-6736(15)60066-2)
- Kirshenbaum, A. P., Olsen, D. M., & Bickel, W. K. (2009). A quantitative review of the ubiquitous relapse curve. *Journal of Substance Abuse Treatment*, 36(1), 8-17. <https://doi.org/10.1016/j.jsat.2008.04.001>
- Klein, A. (2008). *Khat and the creation of tradition in the Somali diaspora*. In: Fountain, S., & Kitzinger, J., (Eds.), *Ethnography and substance use*, 61-78 p., Routledge.
- Kongson, J., Sudsutad, W., Thaiprayoon, C., Alzabut, J., & Tearnbuch, C. (2021). On analysis of a nonlinear fractional system for social media addiction involving Atangana–Baleanu–Caputo derivative. *Advances in Difference Equations*, 2021(1), 1-29. <https://doi.org/10.1186/s13662-021-03515-5>
- Laine, E.-M., Piilo, J., & Breuer, H.-P. (2012). Nonlocal memory effects in the dynamics of open quantum systems. *arXiv preprint arXiv:1111.4481*.
- Lang, Q., & Lu, J. (2024). Learning memory kernels in generalized Langevin equations. *arXiv preprint arXiv:2402.11705*.
- Lesourd, C. (2023). Produce, export but control the khat money (Dire Dawa, Ethiopia). *Cahiers D'études Africaines*, 249, 9-36. <https://doi.org/10.4000/etudesaficaines.40506>
- Magin, R. L. (2006). *Fractional calculus in bioengineering*. Begell House.
- Mainardi, F. (2011). Models based on Mittag-Leffler functions for anomalous relaxation in dielectrics. *European Physical Journal Special Topics*, 193(1), 133-160. <https://doi.org/10.1140/epjst/e2011-01385-8>
- Mains, D. (2019). *Under construction: Technologies of development in urban Ethiopia*. Duke University Press.
- Meerschaert, M. M., Mortensen, J., & Wheatcraft, S. W. (2019). Fractional vector calculus for fractional advection-dispersion. *Physica A: Statistical Mechanics and its Applications*, 367, 53-64. <https://doi.org/10.1016/j.physa.2005.07.019>
- Metzler, R., & Klafter, J. (2000). The random walk's guide to anomalous diffusion: A fractional dynamics approach. *Physics Reports*, 339(1), 1-77. [https://doi.org/10.1016/S0370-1573\(00\)00070-3](https://doi.org/10.1016/S0370-1573(00)00070-3)
- Nigatu, W., Eticha, G., Kinfe, E., Wuletaw, Y., & Mekuriaw, W. (2022). What can research evidence tell us about effect of khat on health and socio-economy? Rapid evidence review. *Ethiopian Journal of Public Health and Nutrition*, 5(2).
- Odenwald, M., Klein, A., & Warfa, N. (2009). Khat use and related problems in Somali combatants. *PLOS Medicine*, 6(12), e1000194. <https://doi.org/10.1371/journal.pmed.1000194>
- Odenwald, M., Hinkel, H., Schauer, E., Neuner, F., Schauer, M., Elbert, T. R., & Rockstroh, B. (2010). The consumption of khat and other drugs in Somali combatants: A cross-sectional study. *PLoS Medicine*, 4(12), e341. <https://doi.org/10.1371/journal.pmed.0040341>
- Odenwald, M., et al. (2009). The consumption of khat and other drugs in Somali combatants: A cross-sectional study. *PLOS Medicine*, 6(12), e1000194. <https://doi.org/10.1371/journal.pmed.1000194>
- Picker, M. J., et al. (2024). A probabilistic model of relapse in drug addiction. *Journal of Mathematical Psychology*, 113, 102844. <https://doi.org/10.1016/j.jmp.2024.102844>
- Podlubny, I. (1999). *Fractional differential equations*. Academic Press.
- Qureshi, S., & Atangana, A. (2019). Fractional modeling for a chemical kinetic reaction in a batch reactor via nonlocal operator with power law kernel. *Physica A: Statistical Mechanics and its Applications*. <https://doi.org/10.1016/j.physa.2019.123950>
- Redish, A. D., Jensen, S., & Johnson, A. (2008). A unified framework for addiction: Vulnerabilities in the decision

- process. *Behavioral and Brain Sciences*, 31(4), 415-437. <https://doi.org/10.1017/S0140525X0800472X>
- Room, R., Fischer, B., Hall, W., Lenton, S., & Reuter, P. (2010). *Cannabis policy: Moving beyond stalemate*. Oxford University Press.
- Rubin, D. C., & Wenzel, A. E. (1996). One hundred years of forgetting: A quantitative description of retention. *Psychological Review*, 103(4), 734-760. <https://doi.org/10.1037/0033-295X.103.4.734>
- Sánchez, F., Wang, X., Castillo-Chávez, C., Gorman, D. M., & Gruenewald, P. J. (2007). Drinking as an epidemic: A simple mathematical model with recovery and relapse. In: *Therapist's guide to evidence-based relapse prevention*, 353-368 p., Academic Press.
- Shafqat, R. (2025). A fractional-order alcoholism model incorporating hypothetical social influence: A theoretical and numerical study. *Journal of Mathematics*. <https://doi.org/10.1155/jom/6773909>
- Sidow, N. O., Gökgül, A., & Hassan, M. S. (2024). Chewing khat (*Catha edulis*) is a risk factor for stroke: A prospective study. *International Journal of General Medicine*, 17, 5707-5713. <https://doi.org/10.2147/IJGM.S482246>
- Stumpf, M. P. H., & Porter, M. A. (2012). Critical truths about power laws. *Science*, 335(6069), 665-666. <https://doi.org/10.1126/science.1216142>
- Valente, T. W. (2012). Network interventions. *Science*, 337(6090), 49-53. <https://doi.org/10.1126/science.1217330>
- Valente, T. W. (2010). *Social networks and health: Models, methods, and applications*. Oxford University Press.
- Valente, T. W., & Pumpuang, P. (2007). Identifying opinion leaders to promote behavior change. *Health Education & Behavior*, 34(6), 881-896. <https://doi.org/10.1177/1090198106297855>
- West, B. J., Grigolini, P., & Bologna, M. (2017). Physics of the human mind. Springer. (Related modeling in West, B. J. et al. works on fractional dynamics; <https://doi.org/10.1016/j.jtbi.2017.05.012>)
- Wixted, J. T., & Carpenter, S. K. (2007). The Wickelgren power law and the Ebbinghaus savings function. *Psychological Science*, 18(2), 133-134. <https://doi.org/10.1111/j.1467-9280.2007.01865.x>
- Wondemagegn, A. T., Cheme, M. C., & Kibret, K. T. (2017). Perceived psychological, economic, and social impact of khat chewing among adolescents and adults in Nekemte Town, East Welega Zone, West Ethiopia. *BioMed Research International*, 7427892. <https://doi.org/10.1155/2017/7427892>
- Wu, L., Sun, P., Fu, Y., Hong, R., Wang, X., & Wang, M. (2019). A neural influence diffusion model for social recommendation. In: Proceedings of the 42nd International ACM SIGIR Conference on Research and Development in Information Retrieval, 235-244 p., ACM.

Funding

Not applicable.

Institutional Review Board Statement

Ethical approval was obtained from the Dire Dawa University Institutional Review Board (Ref: DDU/IRB/2023/042), Dire Dawa University (Protocol No. DDU/IRB/2023/042).

Informed Consent Statement

Not applicable.

Copyrights

Copyright for this article is retained by the author(s), with first publication rights granted to the journal.

This is an open-access article distributed under the terms and conditions of the Creative Commons Attribution license (<http://creativecommons.org/licenses/by/4.0/>).