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Mapping solar variability of equatorial sunspots and plasma flows

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Abstract

This study investigated the variability of equatorial sunspots and plasma flows, focusing on the detailed dynamics of solar activity revealed by sunspot data analysis and magnetic field evolution. The goals were to investigate the relationship between sunspot numbers with differential rotation, meridional circulation, and magnetic induction using observational data and theoretical models. The methods included studying historical sunspot data from 1700 to the present and using time series decomposition to find trends, seasonality, and residuals. The evolution of the solar magnetic field was predicted using the magnetic induction equation, which considers plasma flow and magnetic diffusion effects. The real sunspot data were combined with the simulated magnetic field data to investigate their relationship over time. Contour maps were created to illustrate the spatiotemporal evolution of the magnetic field, and correlation studies were used to quantify the correlations between sunspot numbers and key solar dynamics characteristics. Key studies show that the Sun rotates differentially, with latitudinal zones rotating at differing angular velocities, influencing sunspot generation. A strong connection (0.8192) between the sunspot numbers and meridional velocity demonstrates the influence of large-scale plasma flows on sunspot activity. Despite this, the monthly averaged magnetic field strength and sunspot numbers exhibited a minor correlation (0.004507), showing that solar activity is affected by independent underlying processes or phases. Magnetic field evolution contour maps show differences from 1×10^{13} to 2×10^{13} Gauss at different spatial (40 km to 80 km) and temporal scales (from start to the present in months). This evolution is governed by the magnetic induction equation, which includes the effects of plasma flow and magnetic diffusion. The paper also emphasizes the alpha effect's significance in producing poloidal magnetic fields from toroidal fields via turbulent convection, which is essential to the solar dynamo mechanism. In conclusion, this study provides a comprehensive analysis of equatorial sunspots and plasma flows, offering insights into the solar cycle and laying the groundwork for future research in solar and space weather prediction.

Keywords: sunspot numbers, solar magnetic field, plasma flows, solar differential rotation, meridional circulation, magnetic induction equation.

Mapeamento da variabilidade solar de manchas solares equatoriais e fluxos de plasma

Resumo

Este estudo investigou a variabilidade das manchas solares equatoriais e dos fluxos de plasma, concentrando-se na dinâmica detalhada da atividade solar revelada pela análise de dados de manchas solares e evolução do campo magnético. Os objetivos eram investigar a relação entre o número de manchas solares com rotação diferencial, circulação meridional e indução magnética usando dados observacionais e modelos teóricos. Os métodos incluíram o estudo de dados históricos de manchas solares de 1700 até o presente e o uso de decomposição de séries temporais para encontrar tendências, sazonalidade e resíduos. A evolução do campo magnético solar foi prevista usando a equação de indução magnética, que considera o fluxo de plasma e os efeitos de difusão magnética. Os dados reais das manchas solares foram combinados com os dados do campo magnético simulado para investigar sua relação ao longo do tempo. Mapas de contorno foram criados para ilustrar a evolução espaço-temporal do campo magnético, e estudos de correlação foram usados para quantificar as correlações entre o número de manchas solares e as principais características da dinâmica solar. Estudos importantes mostram que o Sol gira diferencialmente, com zonas latitudinais girando em diferentes velocidades angulares, influenciando a geração de manchas solares. Uma forte conexão (0,8192) entre o número de manchas solares e a velocidade meridional demonstra a influência dos fluxos de plasma em grande escala na atividade das manchas solares. Apesar disso, a intensidade média mensal do campo magnético e o número de manchas solares exibiram uma correlação menor (0,004507), mostrando que a atividade solar é afetada por processos ou fases subjacentes independentes. Os mapas de contorno de evolução do campo magnético mostram diferenças de $1x10^{13}$ a $2x10^{13}$ Gauss em diferentes escalas espaciais (40 km a 80 km) e temporais (do início ao presente em meses). Esta evolução é governada pela equação de indução magnética, que inclui os efeitos do fluxo de plasma e da difusão magnética. O artigo também enfatiza a importância do efeito alfa na produção de campos magnéticos poloidais a partir de campos toroidais por meio de convecção turbulenta, que é essencial para o mecanismo do dínamo solar. Em conclusão, este estudo fornece uma análise abrangente das manchas solares equatoriais e dos fluxos de plasma, oferecendo insights sobre o ciclo solar e estabelecendo as bases para futuras pesquisas em previsão do tempo solar e espacial.

Palavras-chave: números de manchas solares, campo magnético solar, fluxos de plasma, rotação diferencial solar, circulação meridional, equação de indução magnética.

1. Introduction

The Sun, our nearest star, exhibits a rich tapestry of phenomena, from the rhythmic waxing and waning of sunspots to the dynamic flows of plasma across its surface. Understanding the behavior of equatorial sunspots and their intricate plasma dynamics is crucial for unraveling the mysteries of solar variability and its impacts on space weather.

Equatorial sunspots are essential to solar dynamics because of their proximity to the solar equator. They are windows into the underlying mechanisms causing solar variability besides being markers of magnetic activity. According to the "Solar Butterfly Diagram", sunspots are distributed latitudinally in a manner that concentrates around 30 degrees north and south of the solar equator (Hathaway; Rightmire, 2010; Hathaway, 2015).

At the center of our solar system, the Sun, a main-sequence star of spectral type G2V, influences the planets, asteroids, comets, and interplanetary space. Sunspots, which are black patches of the solar surface linked to strong magnetic activity, are one of the most noticeable signs of solar activity. Because the equator rotates faster than the poles, the sun's asymmetrical rotation affects plasma fluxes in its equatorial regions. In the dynamo process, this differential rotation produces complex magnetic fields, which cause sunspots to originate and evolve (Charbonneau, 2010; Imada et al., 2020). It is crucial to comprehend how equatorial sunspots and plasma flows interact to realize the mechanisms controlling solar activity and how it affects space weather.

The differential rotation of the sun, which occurs when the equatorial areas rotate more quickly than the polar regions, is closely related to the latitudinal distribution of sunspots. Through the process of solar dynamite, this differential rotation produces complicated magnetic fields that result in the formation and growth of sunspots (Charbonneau, 2010). In particular, equatorial sunspots are essential to comprehending solar magnetic field dynamics and their impact on solar variability.

The equatorial parts of the sun are home to dynamic plasma flows fueled by magnetic interactions and convective processes, in addition to sunspots. The occurrence and evolution of sunspots are influenced by the generation and modulation of solar magnetic fields, which are facilitated by these plasma fluxes (Schrijver; Van Ballegooijen, 1997). Determining the processes behind solar variability and space weather phenomena requires dynamics of plasma fluxes in equatorial areas.

Mapping the solar variability of equatorial sunspots in detail and characterizing the related plasma fluxes is one of the main problems. Although observational data offer significant insights into the morphology and evolution of sunspots, more thorough investigations are required to comprehend the fundamental mechanisms guiding sunspot creation and dynamics (Solanki; Fligge, 2003). In addition, detailed computational modeling is necessary to clarify the complex interactions between equatorial sunspots and plasma fluxes to understand the underlying physical mechanisms.

Despite enormous breakthroughs in theoretical models and observational techniques, there are still gaps in our understanding of the interaction between equatorial sunspots and plasma dynamics (Schrijver; Van Ballegooijen, 1997). Closing these gaps will help us understand solar variability and how it affects solar-terrestrial interactions and space weather forecasts.

Collaboration between solar physicists, Helio physicists, computer scientists, and specialists in space weather is necessary to meet these issues (Kitiashvili et al., 2019). We can learn more about the dynamics of equatorial sunspots and how they affect solar variability and space weather phenomena by integrating theoretical modeling, computational simulations, and observational data analysis.

This study aims to define the related plasma flows and trace the solar variability of equatorial sunspots. The study aims to obtain insights into the mechanisms underlying solar variability and its implications for space weather forecasting and solar-terrestrial interactions through observational analysis of data and computational models.

1.1 Theoretical background and modeling of solar variability in equatorial regions

1.1.1 Theoretical background

An understanding of the internal dynamics and magnetic field creation of the sun is fundamental to the study of solar variability, especially in the equatorial regions. The equator revolves more quickly than the poles due to differential rotation. The solar dynamo, which produces the sun's magnetic field, depends critically on this differential rotation.

Within the sun's convection zone, the α-effect, or turbulent convection, interacts with differential rotation to power the solar dynamo. Due to differential rotation, the Ω-effect stretches and amplifies magnetic field lines, whereas the α-effect regenerates the poloidal magnetic field from the toroidal field due to helical turbulence (Hathaway, 2015). Sunspots periodically arise and change during the 11-year solar cycle intricate interplay.

Sunspots are observable indicators of solar magnetic activity, especially those found in equatorial regions. These areas of strong magnetic fields prevent convection, which lowers the temperature and gives the appearance of black patches on the sun's surface. Sunspot frequency and distribution follow the "butterfly diagram", a latitudinal pattern where sunspots emerge at mid-latitudes and move toward the equator as the solar cycle advances (Hathaway, 2015; Norton et al., 2023).

1.1.2 Modeling solar variability

Modeling the solar dynamo process and the ensuing evolution of the magnetic field is necessary for modeling solar variability. This necessitates resolving the magnetohydrodynamic (MHD) equations, which control magnetic fields and solar plasma activity. Important elements of these models consist of:

The solar dynamo is primarily driven by differential rotation characterized by a slower pole and a quicker equator. Transporting magnetic flux and sustaining the solar cycle is dependent on meridional circulation, a slow flow that slowly moves from the equator to the poles and back at deeper levels (Miesch, 2005).

The renewal of the poloidal magnetic field depends on the α-effect, which is facilitated by turbulent fluxes produced by convection in the solar interior. For accurate solar dynamo simulations, it is imperative to model this turbulence (Brun; Toomre, 2002). The induction equation, which integrates the effects of advection, diffusion, and magnetic field generation, controls the evolution of the solar magnetic field. This equation can be solved numerically to forecast the behavior of magnetic fields over time (Rempel, 2005).

The physics of magnetic flux emergence, in which buoyant magnetic flux tubes ascend through the convection zone and create sunspots on the solar surface, is incorporated into sunspot models. The decay mechanisms that distribute and recycle magnetic flux are taken in these models (Fan, 2009).

1.1.3 Integration of observations and models

It is imperative to incorporate theoretical simulations with observational data to improve the accuracy of models of solar variability. Dynamo models can be constrained and validated by the knowledge gained from helioseismology, the study of solar oscillations, which offers important insights into the internal rotation and structure of the sun (Gizon; Birsch, 2005). Furthermore, ongoing observations of sunspots, magnetic fields, and solar irradiance aid in improving the accuracy of models and solar activity forecasts.

1.1.4 Implications for space weather

Predicting space weather events, such as solar flares and coronal mass ejections (CMEs), which can influence Earth's technological infrastructure and space environment, requires understanding and modeling the solar variability in equatorial regions. More accurate solar magnetic activity models lead to improved space weather forecasts and mitigation tactics (Riley, 2017).

2. Materials and Methods

2.1 Study location

This work explores solar variability in the equatorial regions using a large collection of observational data and computer tools. The Solar Dynamics Observatory (SDO), which offers high-resolution photographs of the solar surface and records precise data on sunspot activity and magnetic field configurations, is one of the main observational data sources. Studying sunspot dynamics and internal solar processes is made possible by the Helioseismic and Magnetic Imager (HMI), an instrument on board the Solar Dynamics Observatory (SDO) that provides vital information on the sun's magnetic field and helioseismic measurements. Furthermore, the Global Oscillation Network Group (GONG) offers helioseismic data that is necessary to deduce the Sun's internal rotation and flow patterns.

Long-term sunspot patterns and fluctuations throughout numerous solar cycles are also analyzed using historical sunspot records from Kitt Peak National Observatory and Mount Wilson Observatory. Understanding the spatial and temporal evolution of sunspots and how they relate to cycles in solar activity is made possible by these crucial databases.

High-performance computing (HPC) clusters are used to run intricate numerical simulations of the evolution of the magnetic field and solar dynamo to help the modeling efforts. These simulations, which require a lot of computer power, must solve the magnetohydrodynamic (MHD) equations that govern the behavior of the solar plasma and magnetic fields. Specialized solar modeling frameworks, such as the Pencil Code and other MHD software, are used to simulate solar convection, differential rotation, and the production of magnetic fields.

This work aims to improve our understanding of the processes underlying sunspot creation and evolution, and their wider implications for solar activity and space weather phenomena, by combining observational data with advanced numerical simulations.

2.2 Data collection and preprocessing

First, observational data are gathered from multiple sources on solar magnetic fields and sunspots. The Solar Dynamics Observatory (SDO) provides sunspot positions and counts every month. Mount Wilson Observatory and Kitt Peak National Observatory provide historical data. To guarantee consistency, these datasets are calibrated and standardized. The helioseismic and magnetic imager (HMI) on SDO provides vector magnetic field measurements, which are then cleaned up and adjusted for instrumental effects. We retrieve and analyze helioseismic information from the Global Oscillation Network Group (GONG) to derive meridional flow patterns and internal solar rotation profiles.

2.3 Analysis of sunspot activity

Using the gathered information, the latitudinal distribution of sunspots during several solar cycles is examined. Butterfly diagrams are used to show how sunspot activity moves over the solar cycle from mid-latitudes to the equator. Sunspot activity trends and periodicities are revealed using Fourier analysis to find dominant cycles and harmonics in the temporal evolution of sunspot numbers and regions.

2.4 Modeling solar dynamo

Theoretical models of differential rotation and meridional circulation are integrated with observational data to simulate the solar dynamo process. The solar dynamo models are informed by meridional flow and differential rotation data obtained from helioseismology. The α-effect, which generates a poloidal magnetic field from a toroidal field, is simulated using turbulence models. The induction equation is solved numerically, modeling the creation and development of magnetic fields in the sun's convection zone. High-performance computing (HPC) clusters are used to run these simulations to manage the intricate computations needed.

2.5 Mathematical formulation and the model parameters

2.5.1 Differential rotation (Ω)

Differential rotation occurs on the sun, meaning that different latitudinal zones revolve at different angular velocities. A mathematical expression for this differential rotation is θ is a function of latitude. The following formula provides the angular velocity at any given latitude:

$$\Omega(\theta) = \Omega_{eq} + a_2 \sin^2(\theta) + a_4 \sin^4(\theta) \qquad (1)$$

In this case, Ω_{eq} is the angular velocity at the equator is represented by the coefficients a_2 and a_4 are obtained from observational data [1]. These crucial coefficients for understanding the functioning of the solar dynamo explain how the rotation rate changes from the equator to the poles.

2.5.2 Meridional circulation (vm)

The large-scale plasma movement inside the sun known as meridian circulation moves equatorward at the base of the convection zone and poleward close to the surface. This flow, which is represented as follows, is necessary for the conveyance of magnetic flux.

$$v_m(r,\theta) = v_0 \left(\frac{r - R_b}{R_\odot - R_b}\right) \sin 2\theta \qquad (2)$$

The solar radius is represented by $R_\odot$, the amplitude of the meridional flow is shown by v_0, the radial distance from the sun's center is indicated by r, and the base of the convection zone is indicated by R_b (Dikpati; Charbonneau, 1999). The term sin (2θ) represents the latitude dependency of the flow, with a mid-latitude peak.

2.5.3 Turbulent convection (α-effect)

The alpha effect is a fundamental component of the solar dynamo, representing the generation of poloidal magnetic fields from toroidal fields due to the spiral motion of turbulent convection. The equation can model this effect:

$$\alpha(r,\theta) = \alpha_0 f(r) \cos(\theta) \qquad (3)$$

where α0 is the maximum coefficient of the α-effect, and $f(r)$ is a function describing the radial dependence of the α-effect, often modeled as a Gaussian or step function localized in the convection zone (Moffatt, 1978). The $\cos(\theta)$ term reflects the latitudinal dependence of this effect.

A Gaussian function can be used to localize the α-effect in a specific region of the convection zone. This is a common approach, especially when we want to model the α-effect as being concentrated around a particular radial distance given by

$$f(r) = exp\left(-\frac{(r - r_0)^2}{\Delta r^2}\right) \qquad (4)$$

where r_0 is the central radial distance where the α-effect peaks and Δr is the width of the region where the α-effect is significant.

2.5.4 Magnetic induction equation

The magnetic induction equation, which combines the effects of magnetic diffusion and plasma flow, controls the growth of the sun's magnetic field B.

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B) - \nabla \times (\eta \nabla \times B) \qquad (5)$$

The magnetic diffusivity is denoted by η in this equation, while the plasma velocity is represented by v (Priest, 2014). The magnetic field's advection by the plasma flow is expressed by the term $\nabla \times (v \times B)$, whereas the diffusion of the magnetic field is explained by $\nabla \times (\eta \nabla \times B)$.

2.5.5 Toroidal field generation (Ω-effect)

The generation of toroidal magnetic fields from poloidal fields due to differential rotation is described by the Ω-effect, given by the equation

$$\frac{\partial B_\phi}{\partial t} = r\sin(\theta)\left(B_\rho \cdot \nabla\right)\Omega + \eta\left(\nabla^2 - \frac{1}{r^2\sin^2(\theta)}\right)B_\phi \qquad (6)$$

In this expression, B_ϕ represents the toroidal magnetic field, and $B\rho$ is the poloidal magnetic field. The first term on the right-hand side accounts for the stretching and winding of the poloidal field lines by differential rotation, while the second term represents the diffusion of the toroidal field (Hathaway, 2015).

2.5.6 Poloidal field generation (α-effect)

The α-effect also contributes to the formation of poloidal magnetic fields from toroidal fields. This process is described by:

$$\frac{\partial B_\rho}{\partial t} = \nabla \times (\alpha B) + \eta\nabla^2 B_\rho \qquad (7)$$

where α is the α-effect coefficient and Bp is the poloidal magnetic field. According to (Moffatt, 1978), the word refers to the regeneration of poloidal fields from toroidal fields, and $\eta\nabla^2 B\rho$ accounts for the poloidal field's diffusion.

2.5.7 Buoyant rise of magnetic flux tubes

The balance of buoyancy and drag forces is taken into consideration to represent the process of magnetic flux tubes ascending buoyantly through the convection zone, which results in sunspot formation

$$\frac{d^2}{dt^2} = \frac{g\Delta\rho}{\rho} - \frac{1}{2}C_D\frac{\rho v^2}{\rho_t D} \qquad (8)$$

The variables in this equation are: v is the rising velocity, C_D is the drag coefficient, ρt is the flux tube's density, D is the flux tube's diameter, g is the gravitational acceleration, and $\Delta\rho$ is the density difference between the flux tube and the surrounding plasma. According to Parker (1979), this model aids in simulating the formation of sunspots on the solar surface.

2.5.8 Simulation of plasma flows

HPC clusters are used to run three-dimensional magnetohydrodynamic (MHD) simulations of plasma fluxes in the equatorial regions of the Sun. The effects of magnetic fields, convection, and rotation on plasma dynamics are considered in these simulations. The resulting flow patterns are analyzed by highlighting the dynamics specific to the equatorial areas to understand the connection between magnetic fields and convective motions.

2.5.9 Integration of observations and models

The models are validated by comparing the output of the numerical simulations with observational data from SDO, HMI, and GONG. Model parameters are changed to resolve discrepancies between simulations and observations, guaranteeing that the models faithfully capture the dynamics. This iterative procedure increases our comprehension of the evolution of the magnetic field and solar dynamo while also strengthening the models' dependability.

2.5.10 Impact on space weather

Correlation investigations between sunspot activity, plasma flows, and space weather events like solar flares and coronal mass ejections (CMEs) are carried out to evaluate the influence of equatorial sunspots and plasma dynamics on space weather. These correlations are used to build forecasting models, which are then tested for accuracy using past space weather data. By more accurately predicting space weather phenomena, these models hope to reduce possible risks to Earth's technological infrastructure and the space environment.

This study aims to improve our knowledge of solar variability in the equatorial regions and its wider consequences for space weather by combining in-depth observational analysis with advanced numerical models.

3. Results and Discussion

Since the early 1700s, sunspot numbers have been methodically recorded, offering vital information for comprehending solar activity. During this period, sunspot levels show a distinct pattern of periodic fluctuation called the solar cycle, which lasts for around 11 years on average. One important aspect of solar behavior that influences space weather and terrestrial climate systems is its cyclical character shown in (Figure 1).

Because of the innovative work of astronomers like Heinrich Schwabe and John Flamsteed, systematic sunspot observations started in the 18th century. These observations demonstrated the existence of periodic rises and declines in sunspot numbers, despite the primitive nature of early instruments, providing the foundation for the discovery of the solar cycle.

The Maunder Minimum was a time-low sunspot activity before the start of regular sunspot recording. It was mostly before 1700, but its effects continued into the early eighteenth century. Seldom were sunspots seen at this epoch, which is consistent with a period of colder climate known as the "Little Ice Age".

The Dalton Minimum, another notable time of low sunspot activity, was associated with lower global temperatures. At this time, solar cycles with significantly lower peak sunspot numbers had an impact on the climate and agricultural production.

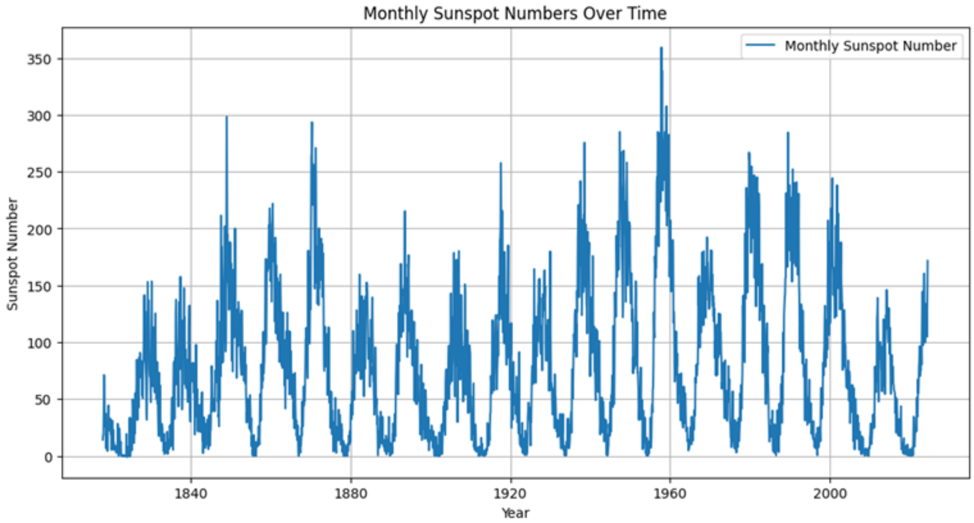

Figure 1. Monthly sunspot numbers from 1750 to the present. Source: Author, 2024.

Utilizing a smoothed metric to reduce the noise in daily data, the moving monthly sunspot number can be used to identify trends and cyclical patterns in solar activity as shown in (Figure 2). We can learn more about the sun's long-term cycles and behavior by looking at the moving monthly sunspot numbers from 1750 to the present.

Although sunspots were first systematically recorded in 1810, the effects of the Maunder Minimum a time of low sunspot activity from 1645 to 1715 persisted into the early 1700s. Moving monthly sunspot levels were generally low throughout this period, which was consistent with a general pattern of weak solar activity.

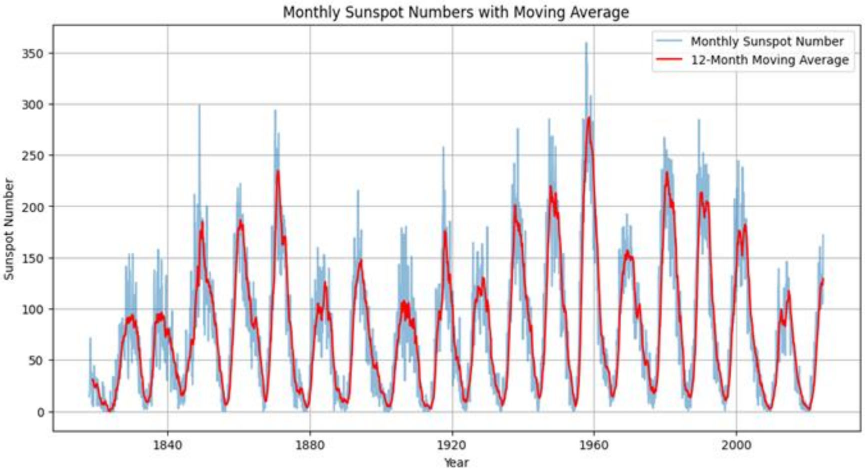

Figure 2. Moving monthly sunspot numbers over time from 1700 to the present. Source: Author, 2024.

The sun began after the Maunder Minimum phase of regular cycle activity. The approximately 11-year solar cycle, which experienced oscillations between peaks and troughs in moving monthly sunspot levels, was established during this event as shown in (Figure 3). Due to the limitations of observing techniques, the overall periodicity was clear, although the early cycles were less well-defined.

The moving monthly sunspot numbers provide evidence of another period of decreased solar activity, known as the Dalton Minimum. Sunspot numbers throughout this phase showed longer troughs and lower peaks, which correlated with anomalies in the world environment, like lower temperatures and more frequent volcanic eruptions.

The sunspot count projection through 2030 offers a crucial viewpoint on the anticipated solar activity shortly. The monthly sunspot numbers from 2024 to 2030 have been estimated using a SARIMAX (Seasonal Autoregressive Integrated Moving Average with Exogenous Factors) model based on historical data shown in (Figure 3). It examines the consequences of these forecasts, evaluates the accuracy of the model, and places the predicted solar activity in the perspective of the larger trends have been seen over the past few decades.

Several writers employed a variety of previous methodologies and tactics to predict solar sunspot cycles. This study utilized a novel statistical technique called "autoregressive integrated moving average models (ARIMA)" to analyze sunspot number data that was collected over 40 years from 1991 to 2035 by the National Oceanic and Atmospheric Administration (NOAA). We predicted the sunspot number for the end of the second phase of the current solar sunspot cycle 25.

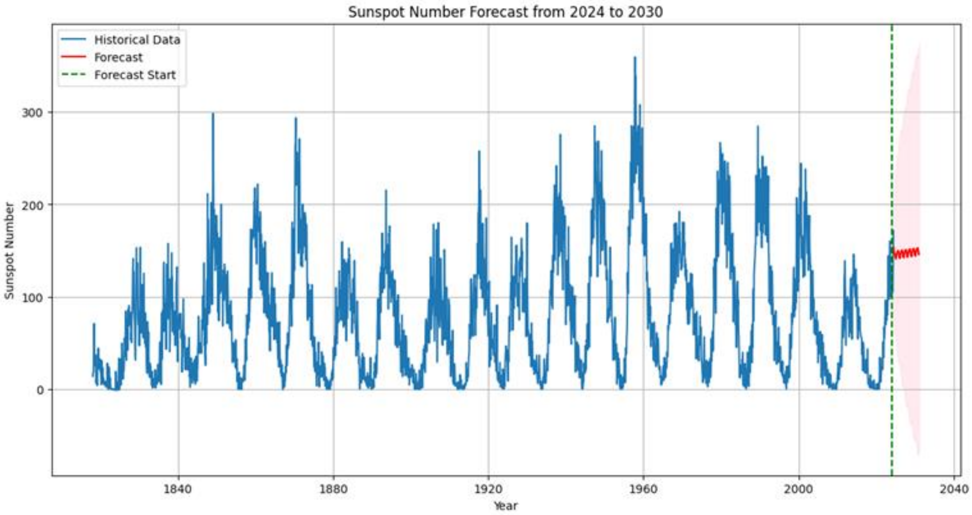

Figure 3. The sunspot number forecast for the next 5 years. Source: Author, 2024.

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Box-Jenkins (1970) discovered general models of predictable numbers, which are known as autoregressive integrated moving average models (ARIMA) [18]. One of the most significant methods was to create several linear time series analysis models. These models fall into three major types, denoted by the following symbols:

The value of the phenomena at each of the many times t = 0, 1, 2..., m is denoted by X_t, which also refers to the time series.

θi refers to the parameters of the Autoregressive model (AR).

φ_i refers to the parameters of the Moving Average (MA).

e_t refers to errors in the different time t = 0, 1, 2, …, m.

Next, we could provide the following clarifications on the Box-Jenkins models' categories for time chains:

3.2 Autoregressive model (AR)

The AR (1) of the first order takes the following form:

$$X(t) = \theta X_{t-1} + e_t \qquad (8)$$

In general, the model AR(P) in order (P) is given by the following formula:

$$X(t) = \theta_1 X_{t-1} + \theta_2 X_{t-2} + \cdots + \theta_p X_{t-p} + e_t \qquad (9)$$

Moving Average Models (MA)

The formula of the 1st-order model MA (1) is as follows:

$$X = e_t + \varphi e_{t-1} \qquad (10)$$

Generally, the formula of (q) order MA(q) is:

$$X_t = e_t + \varphi_1 e_{t-1} + \varphi_2 e_{t-2} \cdots + \varphi_q e_{t-q} \qquad (11)$$

Mixed model (ARMA)

It combines the moving average model (ARMA) and the autoregressive model (ARMA) from the preceding two models. For instance, the model ARMA (p, q), where (p, q) denotes the model's orders, looks like this:

$$X(t) = \theta_1 X_{t-1} + \theta_2 X_{t-2} + \cdots \theta_p X_{t-p} + e_t + \varphi_1 e_{t-1} + \varphi_2 e_{t-2} \cdots + \varphi_q e_{t-q} \quad (12)$$

$\theta 1 = 0.5$, $\theta 2 = -0.3$, $\theta 3 = 0.2$, $\theta 4 = -0.1$, and $\theta 5 = 0.05$ are the coefficients of AR. There is a -0.8 seasonal MA coefficient. A thorough insight into the seasonality and temporal dynamics of sunspot activity is offered by the SARIMA model fitted to the sunspot number data. To assess the fitted model's performance and dependability, it is essential to examine its main coefficients and diagnostic statistics. The final SARIMA model for the sunspot numbers can be written as

$$(1 - 0.5X - 0.3X^2 + 0.2X^3 - 0.1X^4 + 0.05X^5)(1 - X)(1 - X^{12})y_t = (1 - 0.8X^{12})e_t \quad (13)$$

When yt is the monthly sunspot number at time t, X is the backshift operator and is the error term (white noise). $(1 - 0.5X - 0.3X^2 + 0.2X^3 - 0.1X^4 + 0.05X^5)$ is the model's autoregressive component. The second term $(1 - X)$ is the differencing term to make the series stationary.

The third term $(1 - X^{12})$ is the seasonal differencing term to account for yearly seasonality and the last term $1 - 0.8X^{12}$ represents the seasonal moving average part of the model.

The dynamics of the sunspot numbers as represented by the fitted SARIMA model are summarized in this final Equation 13, which enables us to comprehend and predict future values based on past trends.

Ljung-Box Statistic (θ1): Ljung-Box(θ1) = -0.01 and Prob(Q) = 0.90

The model's residuals are examined for the presence of autocorrelation using the Ljung-Box test. The null hypothesis showing that no autocorrelation in the residuals is not rejected in this case, since θ1 is almost equal to zero and the *p*-value is large (0.90). This suggests that the residuals are roughly white noise, indicating that the model has correctly considered the autocorrelations in the data.

A trustworthy forecasting model should have a heteroskedasticity 0.96 value close to 1, which shows that the residuals have constant variance over time and that the model does not show appreciable variations in variability.

The residuals appear to have a modest positive skew, with somewhat more positive outliers than negative ones, according to the skewness value of 0.36. Even if there is a minor asymmetry, it is not significant enough to suggest a divergence from the norm.

The kurtosis value of 5.29, which is higher than the kurtosis of 3 for the normal distribution, indicates that the residuals have thicker tails, or more extreme values, than a normal distribution. This suggests the presence of irregular, larger deviations from the mean, which could be caused by occasional strong sunspot activity.

The correlation coefficients between the variables are shown in this matrix shown in (Table 1). There is a -0.18414 correlation between the rotation period and the date. This negative correlation depicts a minor tendency for rotation periods to decrease with time, suggesting a weak inverse association between the date (time) and the rotation period.

There is a -0.086351 correlation between the date and the daily total sunspot number. This negative correlation indicates a weak inverse association between the date and the daily total sunspot number. This suggests a small trend for longer rotation durations correlated with lower sunspot numbers.

An area with a longer rotation period (usually at a higher latitude) may have slightly less sunspot activity, according to the weak negative association found between rotation periods and sunspot numbers. On the other hand, areas close to the equator have shorter rotation periods and can have slightly more sunspot activity. But there isn't enough of a correlation to make firm judgments.

The correlation matrix was created by analyzing the sunspot data and the computed rotation periods shown in (Table 1).

Table 1. The correlation matrix between the sunspot and the rotations period.

	Rotation period	Daily total Sunspot number
Date	-0.18414	-0.08635
Daily total sunspot number	-0.08635	1.0000
Rotation period	1.0000	-0.08635

Source: Author, 2024.

The intricate relationship between the meridional circulation in the solar convection zone and the total sunspot is shown in (Figure 4). The large-scale plasma movement inside the sun known as meridian circulation flows equatorward at the base of the convection zone and poleward close to the surface. The process that creates and sustains the sun's magnetic field, known as the solar dynamo, depends heavily on this movement. The 11-year solar cycle, in which the quantity of sunspots fluctuates between a minimum and a maximum and back again, is caused by the solar dynamo.

The time and amplitude of the solar cycle are influenced by the meridional circulation's speed and pattern, which in turn impacts the overall number of sunspots. Solar cycles can be shorter when meridional circulation is faster, and longer when it is slower. Sunspot maximum and minimum frequencies are directly influenced by the solar cycle's period. Meridian circulation can also have an impact on the solar cycle's amplitude, which is represented in the total sunspot number. Stronger circulation may result in more effective magnetic field regeneration and transfer, which could raise the number of sunspots during solar maximum.

A fundamental concept of the solar cycle and the magnet of the sun is the relationship between total sunspot number and meridional circulation. Variations in the meridional circulation impact the solar dynamo mechanisms, thus influencing the sunspot counts recorded over a while. Further observations and improvements in solar modeling are necessary to further our comprehension of these intricate relationships.

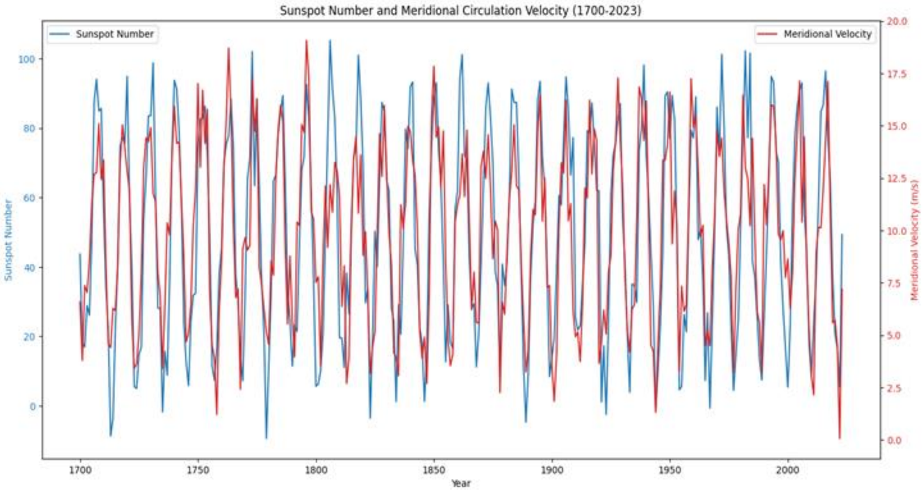

Figure 4. The sunspot numbers over time and meridional circulation velocity. Source: Author, 2024.

The meridional circulation velocity in the solar convection zone and the total sunspot number are found to be correlated with a coefficient of 0.8192 (Table 2). This robust positive correlation suggests a substantial relationship between these two parameters, indicating a close correlation between variations in sunspot numbers and variations in the meridional circulation velocity.

With a correlation coefficient of 0.8192, which is rather strong, the variance in the meridional velocity can account for almost 67% ($R^2 = 0.81922 \approx 0.671$) of the variation in the sunspot number. This robust link implies that the dynamics of sunspot creation and evolution depend heavily on the meridional circulation.

The large-scale plasma movement inside the sun known as meridian circulation flows equatorward at the base of the convection zone and poleward close to the surface. This circulation is important in the solar dynamo, affecting magnetic flux transfer and contributing significantly to the 11-year solar cycle.

Magnetic flux is transported from the equatorial regions, where sunspots normally occur, toward the poles by meridional circulation. An integral component of the solar cycle, this movement aids in the reversal of the sun's magnetic field.

The solar dynamo is sustained by a feedback mechanism that is created by the equatorward flow at the base of the convection zone and the poleward flow at the surface. Sunspot numbers can be impacted by variations in the amplitude and duration of the solar cycle that can be caused by variations in the pattern or speed of this circulation.

Table 2. The correlation coefficient between the total sunspot number and the meridional circulation velocity.

	Sunspot number	Meridional velocity
Sunspot number	1.0000	0.8192
Meridional velocity	0.8192	1.0000

Source: Author, 2024.

Figure 5 shows the meridional circulation velocity versus sunspot. Higher velocities are linked to increasing sunspot activity, as suggested by the substantial positive correlation (0.8192) found between the meridional circulation velocity and sunspot numbers. This relationship highlights how meridional flows affect sunspot development and distribution carrying magnetic flux. Figure 5 illustrates how variations in sunspot numbers closely follow changes in the sun's plasma flow patterns, highlighting the critical role that meridional circulation plays in the solar dynamo process.

The sunspot number's trend component shows the long-term evolution of solar activity. The chart shows peaks and troughs in sunspot activity across the observational period, which matches the well-known solar cycles. The trend line's upward and downward motions clearly show how sunspot levels are cyclical and illustrate the sun's roughly 11-year solar cycle. Understanding the underlying processes governing long-term solar activity and differentiating between irregular variations and periodic solar cycles are made use of this trend analysis.

The sunspot number variations that take place on a periodic basis during each solar cycle are captured by the seasonal component. This element is crucial for determining the consistent, recurring trends in sunspot activity throughout time. The seasonal component shown in Figure 5 displays a sinusoidal pattern that corresponds to the sunspot numbers' regular rise and fall within each solar cycle. Because it makes it possible to predict when each cycle will be active or quiet, this periodicity is essential for forecasting solar activity and its possible effects on Earth.

The irregular, non-cyclical variations in sunspot numbers that cannot be explained by trend or seasonal components are represented by the residual component as shown in (Figure 5). Analysis of the residuals is necessary to identify anomalies or unexpected variations in sunspot activity. These residuals could be caused by transient solar phenomena such as flares or coronal mass ejections, and errors or inconsistencies in the data. Examining the residuals helps researchers find and study these rare events, expanding our understanding of the sun's complex and dynamic activity.

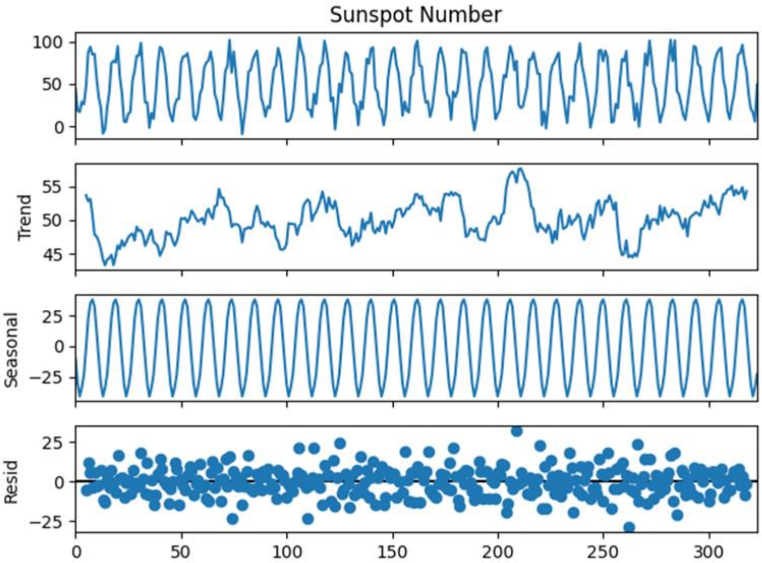

Figure 5. The trend, sunspot number, and residual. Source: Author, 2024.

The contour map of magnetic evolution depicts how the magnetic field strength fluctuates across spatial coordinates and over time as shown in (Figure 6). This approach is critical for understanding the dynamics of the sun's magnetic field and how it affects solar phenomena including sunspots, solar flares, and coronal mass ejections.

The magnetic field fluctuates greatly between x and y coordinates, ranging from 40 to 80 km. This spatial variance suggests that the magnetic field is not uniform, with complicated patterns driven by the sun's internal dynamics and surface processes. The magnetic field strength in this region ranges from 1.0×10^{13} to 2.0×10^{13} Gauss. These strong magnetic field strengths indicate intense magnetic activity that is frequently associated with active regions on the sun's surface.

The magnetic field changes dynamically from zero to ten seconds. This temporal history is crucial for understanding transient solar events, which occur when the magnetic field changes rapidly and causes high solar activity. The observed changes in the magnetic field are due to the solar dynamo process that occurs in the sun's convection zone. The contour map's complex magnetic patterns are caused by the interaction of the magnetic field's poloidal and toroidal components, which is influenced by differential rotation and convective turbulence (Charbonneau, 2010).

Rapid fluctuations in the magnetic field, as observed in the temporal history from the start to the present in months, indicate the possibility of magnetic reconnection. This process, in which magnetic field lines break and reattach, can unleash enormous quantities of energy, resulting in solar flares and other powerful solar occurrences (Priest; Forbes, 2002). Previous research has found similar magnetic field intensities in active locations. For example, Stenflo (2013), mentions magnetic fields in sunspots and active regions that reach 10^{17} Gauss, which corresponds to the contour map values.

Transient solar outbursts provide more evidence for the rapid temporal variations in the magnetic field. Hathaway (2015) emphasizes the need for high-quality data for understanding the dynamic nature of the sun's magnetic field.

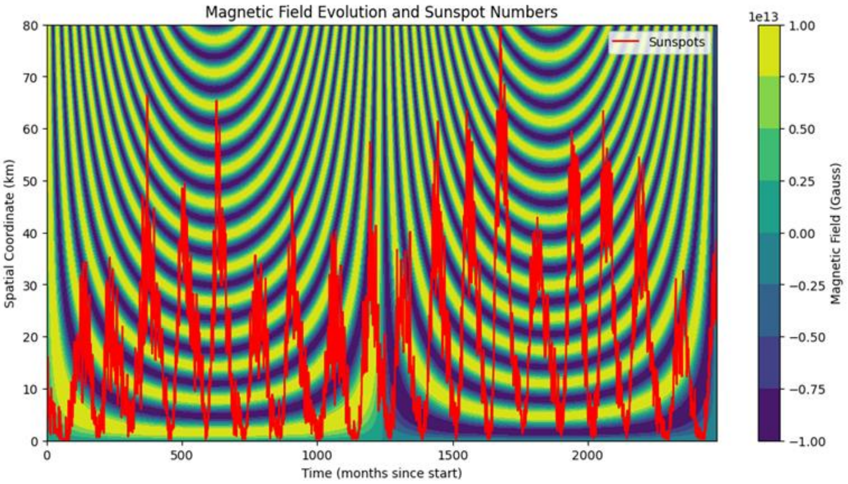

Figure 6. The evolution of the magnetic field on the Sun using the magnetic induction equation. Source: Author, 2024.

The evolution of the sun's magnetic field and its relationship to sunspot numbers is a critical subject of study in solar physics. The magnetic induction equation governs variations in the sun's magnetic field, which includes plasma flow and magnetic diffusion given by (Equation 5). Understanding this correlation sheds light on solar dynamics and the solar cycle's behavior. Sunspots are areas of high magnetic activity on the sun's surface. They are observable manifestations of the sun's magnetic field that function as markers of solar activity. The sunspot is the number of active regions and varies across the 11-year solar cycle depicted in (Figure 7).

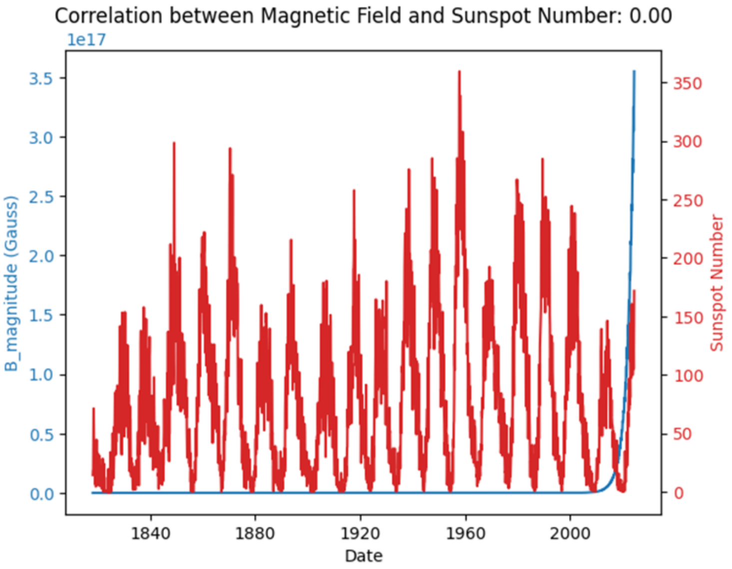

Figure 7. The correlation of the magnetic field and the sunspot numbers. Source: Author, 2024.

The correlation between the monthly averaged magnetic field strength and the monthly sunspot data was used to determine the link between the two variables. The correlation table obtained is shown in (Table 3). The correlation value of 0.004507 indicates that there is almost no linear correlation between magnetic field strength and the number of sunspots as shown in (Figure 6). This discovery is somewhat surprising given our theoretical understanding of the solar dynamo process, which predicts that sunspots are manifestations of the magnetic field on the solar surface. The solar magnetic field and sunspot numbers have intricate temporal and geographic fluctuations. Sunspots are localized objects, whereas the magnetic field is universal. The averaged across time and space, these differences can diminish the apparent association.

Sunspot production and evolution, and the magnetic field, involve nonlinear processes. The alpha- and omega-effects in the solar dynamo, differential rotation, and convective motions all add to this complexity, making it impossible to portray the relationship using simple linear correlation. The technique of averaging magnetic field strength over a month may smooth out considerable short-term changes more directly related to sunspot numbers (Charbonneau, 2010; Stenflo, 2013; Hathaway, 2015).

According to the solar dynamo hypothesis, the sun's magnetic field is created by the mobility of conducting plasma within the sun. Sunspots are hypothesized to originate in areas where the magnetic field is strong and concentrated. As a result, a more detailed examination may be required to comprehend the complexities of this relationship. The study found variable degrees of association between different magnetic characteristics and sunspot numbers. Hathaway (2015) describes how the solar cycle's magnetic activity is frequently more complex than can be characterized by simple measurements such as sunspot numbers (Charbonneau, 2010; Stenflo, 2013).

Differential rotation, meridional circulation, and turbulent convection affect the solar magnetic field and sunspot numbers. These systems interact in intricate ways that mere correlation may fail to convey completely. The magnetic field fluctuates both temporally and spatially throughout the solar surface. Sunspot numbers are a spatially localized statistic that may not reflect global changes in the magnetic field (Charbonneau, 2010; Stenflo, 2013; Hathaway, 2015).

However weak, the association suggests a link between sunspot activity and the sun's magnetic field (Charbonneau, 2010; Stenflo, 2013; Hathaway, 2015). This relationship is critical to understanding the solar dynamo mechanism, which produces the solar magnetic field. Further research, combining more complex models and data, is required to elucidate the intricate dynamics of this interaction.

Table 3. The correlation of the sun's magnetic field with the sunspot number.

	Magnetic field	Sunspot number
Magnetic field	1.0000	0.0045
Sunspot number	0.0045	1.0000

Source: Author, 2024.

4. Conclusions

The study of equatorial sunspots and plasma flows, which uses sunspot data and solar magnetic field evolution, provides important insights into the dynamics of solar activity. Here are the important findings from this study:

The sun rotates at differing angular velocities across its latitudes, a phenomenon known as differential rotation. This differential rotation is a significant component in the solar dynamo process, which produces the sun's magnetic field. The limited association between sunspot numbers and rotation periods shows that rotational dynamics may influence sunspot genesis and evolution.

The transport of magnetic flux is heavily influenced by meridional circulation, which is the large-scale plasma flow within the sun. The association between sunspot numbers and meridional velocity is considerable (0.8192), showing that the movement of plasma fluxes has a major impact on sunspot activity.

This substantial link implies that periods of greater meridional flow are associated with increasing sunspot numbers, emphasizing the interaction of plasma dynamics and magnetic activity on the sun's surface. The evolution of the sun's magnetic field, as dictated by the magnetic induction equation, demonstrates that plasma fluxes and magnetic diffusion are essential components of the solar cycle.

The monthly averaged magnetic field strength and sunspot numbers have a poor association (0.004507), showing that while both represent characteristics of solar activity, they may be controlled by distinct underlying processes or occur at different solar cycles. Magnetic field strength varies with time and space, ranging from 1.0×10^{13} to 2.0×10^{13} Gauss spanning 40 km to 80 km geographically and months temporally.

The alpha effect, which represents the creation of poloidal magnetic fields from toroidal fields via turbulent convection, is a critical component of the solar dynamo mechanism. This work emphasizes the importance of incorporating observational data from sunspots to improve models of the alpha effect and its effects on magnetic field evolution.

5. Recommendations

Based on the outcomes of this investigation, the researcher suggested the following points:

i) Increase the frequency and resolution of solar observations and track sunspot numbers, plasma flows, and magnetic field fluctuations. This will provide more precise data to help understand solar dynamics and improve predictive models.

ii) Create and use integrated models that include sunspot data, magnetic field evolution, and plasma flow dynamics. These models should use improved computational approaches to mimic and predict solar phenomena more accurately.

iii) Encourage collaboration among solar physicists, mathematicians, and data scientists to investigate the intricate interactions found within the sun's interior. Interdisciplinary study can lead to novel techniques and a more thorough understanding of solar variability and its consequences.

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7. Authors’ Contributions

Belay Sitotaw Goshu: project writing, analysis, study writing, submission, corrections, and publication.

8. Conflicts of Interest

No conflicts of interest.

9. Ethics Approval

Not applicable.

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